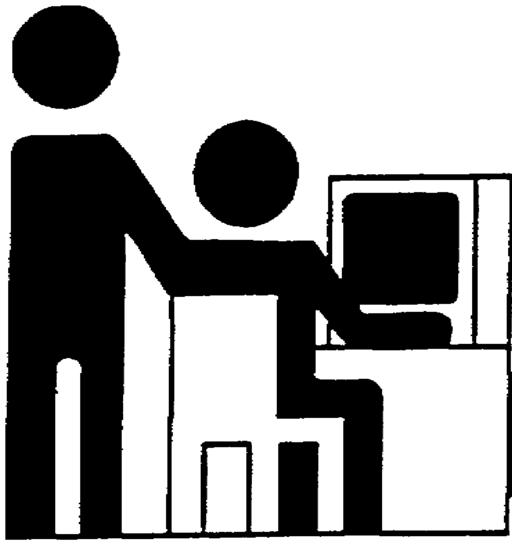
SUBCOURSE
OD1640

EDITION
7

MACHINE SHOP CALCULATION

THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT

ARMY CORRESPONDENCE COURSE PROGRAM



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US ARMY REPAIR SHOP TECHNICIAN WARRANT OFFICER
ADVANCED CORRESPONDENCE COURSE

MOS/SKILL LEVEL: 441A

MACHINE SHOP CALCULATION

SUBCOURSE NO. OD1640

US Army Correspondence
Course Program

11 Credit Hours

GENERAL

The purpose of this subcourse is to introduce various mathematical calculations involved in the machine shop operations of a maintenance company organization in the field.

The scope of the subcourse serves to introduce the methods and procedures for solving problems involving addition, subtraction, multiplication, and division of fractions and decimals, and conversion of fractions to decimals and decimals to fractions; conversion of linear measurements from the English to the metric system and vice-versa; and for solving problems using ratio, proportion, and trigonometry.

Eleven credit hours are awarded for successful completion of this subcourse.

Lesson 1: ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF FRACTIONS AND DECIMALS; AND CONVERSION OF FRACTIONS TO DECIMALS AND DECIMALS TO FRACTIONS

TASK 1: Describe the processes for adding, subtracting, multiplying, and dividing fractions.

TASK 2: Describe the processes for converting fractions to decimals and decimals to fractions; and for adding, subtracting, multiplying, and dividing decimals.

Lesson 2: CONVERSION OF LINEAR MEASUREMENTS FROM THE ENGLISH TO THE METRIC SYSTEM AND FROM THE METRIC TO THE ENGLISH SYSTEM; AND SOLVING PROBLEMS USING RATIO, PROPORTION, AND TRIGONOMETRY

TASK 1: Describe the processes for converting linear measurements from the English to the metric system and from the metric to the English system.

TASK 2: Describe the processes for solving problems using ratio and proportion.

TASK 3: Describe the processes for solving problems using trigonometry.

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***** IMPORTANT NOTICE *****

THE PASSING SCORE FOR ALL ACCP MATERIAL IS NOW 70%.

PLEASE DISREGARD ALL REFERENCES TO THE 75% REQUIREMENT.

When used in this publication “he,” “him,” “his,” and “men” represent both the masculine and feminine genders, unless otherwise stated.

STUDENT NOTES

LESSON 1

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF FRACTIONS AND DECIMALS; AND CONVERSION OF FRACTIONS TO DECIMALS AND DECIMALS TO FRACTIONS

TASK 1. Describe the processes for adding, subtracting, multiplying, and dividing fractions.

CONDITIONS

Within a self-study environment and given the subcourse text, without assistance.

STANDARDS

Within two hours

REFERENCES

No supplementary references are needed for this task.

1. Introduction

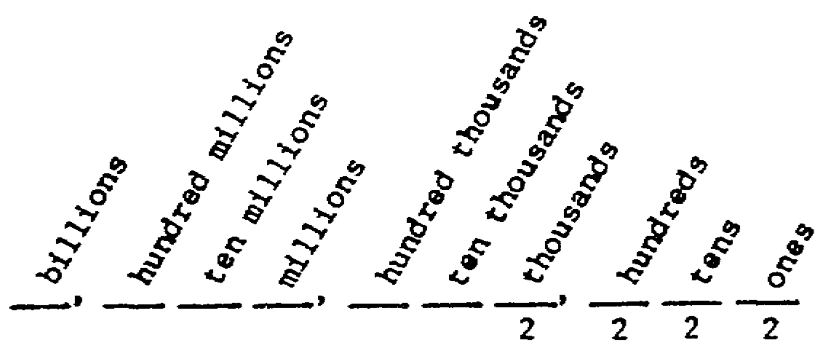
The service section of a ground equipment maintenance support company organization contains a machine shop for support of the company's repair shop elements and supported units. This support consists of refurbishing repair parts, and designing and fabricating jigs, fixtures, and special tools, through the use of a lathe, shaper, and/or milling machine. Such machines provide machine shop personnel with the capability for fabricating component parts with the correct dimensions and with tolerances down to thousands of an inch clearance between moving parts, allowing their efficient operation.

Determining these dimensions and close tolerances, requires the performance of certain mathematical calculations involving a knowledge of how to solve fractions. Subsequent paragraphs, therefore, describe the processes involved in adding, subtracting, multiplying, and dividing fractions. Before delving into fractions, a review of the whole number system will be provided, including addition, subtraction, multiplication, and division.

2. Review of Whole Numbers

a. *Definition*. Whole numbers are made up of the digits 0 through 9. The number 2,222, for example, has four digits. Each digit has a different value because of its position in the number. Figure 1 provides the names of the first ten places in the whole number system.

FIGURE 1. PLACE VALUE OF WHOLE NUMBERS.



Reading from left to right, the first 2 in the number 2,222 has a value of 2 thousands or 2,000. The second 2 has a value of 2 hundreds or 200. The third 2 has a value of 2 tens or 20, and the fourth 2 has a value of 2 ones or 2. The comma makes large numbers easier to read. The number 2,222 would be read as two-thousand, two-hundred, and twenty-two.

b. *Addition*. A requirement for addition is indicated by the symbol +. The answer to addition problems is called the sum or total. To find the sum or total of two numbers, each one having one or more digits, place each digit in its corresponding value position under the other. Put the ones under the ones, and tens under the tens, and so on. Follow the procedure shown in Example 1 on the following page.

EXAMPLE 1. Add 128 to 475.

```
 475
+128
----
```

Step 1. Add the digits in the ones position. 5 + 8 = 13. Write 3 under the ones position. Carry over the 1 to the tens column.

```
   1
 475
+128
----
   3
```

Step 2. Add the digits in the tens position. 1 + 7 + 2 = 10. Write the 0 under the tens position. The 1 is carried over to the hundreds position.

```
 1
 475
+128
----
  03
```

Step 3. Add the digits in the hundreds position. 1 + 4 + 1 = 6.

```
 475
+128
----
 603
```

Step 4. Check by adding from the bottom up following the procedure stated in steps 1 through 3.

c. *Subtraction*. A requirement for subtraction is indicated by the symbol -. The answer to subtraction problems is called the difference. To find the difference between two numbers, write the numbers in columns under each other, as in addition, and proceed as shown in Example 2.

EXAMPLE 2. Subtract 25 from 78.

```
 78
-25
---
```

Step 1. Subtract the digits in the ones position. 8 - 5 = 3.

```
 78
-25
---
  3
```

Step 2. Subtract the digits in the tens position. 7 - 2 = 5.

```
 78
-25
---
 53
```

Step 3. Check by adding the answer to the bottom number. 53 + 25 = 78.

d. *Multiplication*. A requirement for multiplication is indicated by the symbol x. The answer to multiplication problems is called the product. To find the product write the numbers in columns under each other, as in addition, and proceed as shown in Example 3.

EXAMPLE 3. Multiply 25 by 3.

```
  25
 x 3
 ---
```

Step 1. Multiply the digits in the ones position. 3 x 5 = 15. Write the 5 under the ones position and carry the 1 to the tens position.

```
  1
  25
 x 3
 ---
   5
```

Step 2. Multiply the digit 2 in the tens column by the digit 3 and add the digit 1 carried over to the tens column. 2 x 3 = 6 + 1 = 7.

```
  25
 x 3
 ---
  75
```

Step 3. Check by dividing the product 75 by 3 as explained in the succeeding paragraph.

e. *Division*. A requirement for division is indicated by the symbol ÷. The answer to division problems is called the quotient. To find the quotient, proceed as shown in Example 4.

EXAMPLE 4. Divide 75 by 3.

```
3)75
```

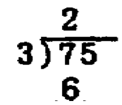
Step 1. Divide the digit 7 by the divisor 3. 7 + 3 = 2. Write the 2 above the 7 in the tens position.

```
  2
3)75
```

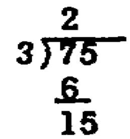
Step 2. Multiply the divisor 3 by the digit 2. 3 x 2 = 6. Write the 6 under the 7.

```
  2
3)75
  6
```

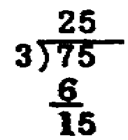
Step 3. Subtract the digit 6 from the digit 7. 7 - 6 = 1. The remainder is 1. The remainder must be less than the divisor 3.



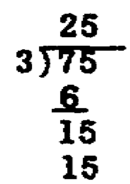
Step 4. Since the remainder is less than the divisor 3, bring down the next digit, 5.



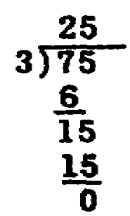
Step 5. Divide the number 15 by 3. 15 ÷ 3 = 5. Write the 5 above the 5 in ones position.



Step 6. Multiply the divisor 3 by the digit 5. 3 x 5 = 15. Write the number 15.



Step 7. Subtract the number 15 from 15. 15 - 15 = 0, indicating that the calculation is complete.



Step 8. Check by multiplying the quotient by the divisor. 25 x 3 = 75.

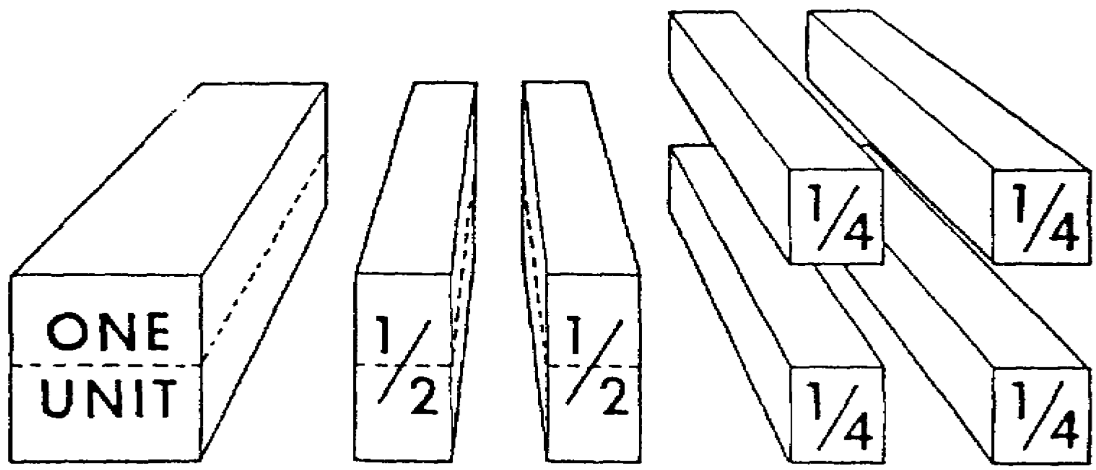
3. Common Fractions

a. *General*. A fraction is any part of an object or number. For example, figure 2 on the following page shows a block of wood as one unit. When it is cut in half, each piece becomes a fraction of the original unit, 1 of 2 equal parts, or 1/2 of the original unit. Cut each of these halves in half again and each piece is now 1 of 4 equal parts, or 1/4 of the original unit.

b. *Definitions*.

(1) *Fraction*. A fraction is an indicated division. It expresses one or more of the equal pieces or parts into which something has been divided.

FIGURE 2. FRACTIONAL PARTS OF A UNIT.



(2) *Terms of a Fraction*. The numerator and the denominator are called the terms of the fraction. The terms are indicated by a diagonal line (/).

(a) *Numerator*. The number above the diagonal line is the numerator. It indicates the quantity of equal parts to be considered.

(b) *Denominator*. The number below the diagonal line is the denominator. It indicates the quantity of equal parts into which the whole unit has been divided.

(3) *Common Denominator*. When two or more fractions have the same denominator, such as 2/5 and 4/5, the 5 is known as the common denominator.

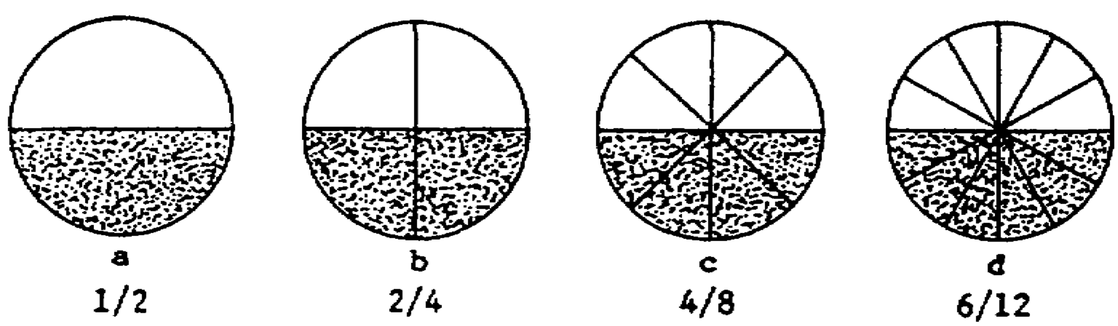
(4) *Least Common Denominator (LCD)*. The LCD of two or more fractions is the least common multiple of the denominators of all the fractions under consideration. For example in 5/7, 1/7, 3/7, the 7 is the LCD. When the denominators are not the same, as in 3/4, 2/5, and 7/10, the LCD is 20, because 20 is the smallest number containing 4, 5, and 10 a whole number of times.

(5) *Proper Fraction*. This is a fraction having a numerator less than the denominator. In other words it is a true fraction of a single whole, such as 9/10 or 7/8 or 21/23.

(6) *Improper Fraction*. This is a fraction having a numerator that is equal to or greater than its denominator. It is either a whole number or a whole number and a fraction of another whole number. For example, 4/4 may be expressed as 1, or 3/2 may be expressed as 1 + 1/2.

(7) *Value of a Fraction*. This is the number that the fraction represents. From the diagrams in figure 3, it can be readily seen that 1/2 in a, 2/4 in b, 4/8 in c, and 6/12 in d are all equal in value, even though the terms of each succeeding fraction are greater than the terms of all the previous fractions. But if the fraction 6/12 was increased by the fraction 1/12, then 6/12 plus 1/12 would equal 7/12, and the value of 7/12 would be greater than the value of all the previously mentioned fractions.

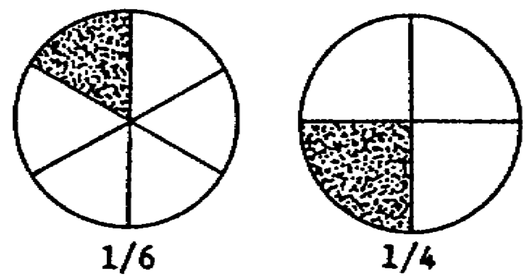
FIGURE 3. VALUE OF A FRACTION.



4. Comparison of Values

If you see two fractions such as 1/4 and 1/6, which is larger? The illustration in figure 4 on the following page shows that 1/4 is the larger fraction.

FIGURE 4. 1/4 IS LARGER THAN 1/6.



a. *Rule 1.* When two fractions have equal numerators and denominators they are of equal value.

EXAMPLE

3/4 and 3/4 are equal fractions.

b. *Rule 2.* When two fractions have equal denominators, the fraction having the larger numerator is the greater in value.

EXAMPLE

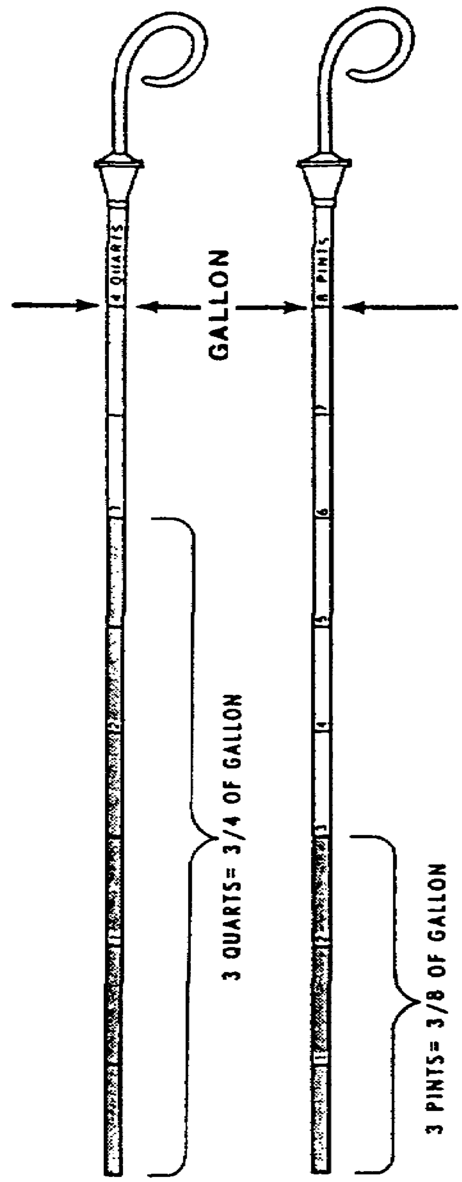
3/7 and 4/7. 4/7 is the larger fraction as it represents 4 of 7 equal parts, whereas 3/7 represents only 3 of 7 equal parts.

c. *Rule 3.* When two fractions have the same numerator, the fraction having the larger denominator is always the smaller.

EXAMPLE

3/4 and 3/8. 3/8 is a smaller fraction than 3/4. 3/8 is only 3 of 8 equal parts, whereas 3/4 is 3 of 4 equal parts, as shown in figure 5 on the following page.

FIGURE 5. FRACTIONAL PARTS OF A GALLON.



5. Reducing Fractions

a. *Reducing to Lowest Terms*. It is sometimes advisable to change a fraction from one form to another without changing its value. This is called reducing the fraction to its lowest terms.

(1) *Rule 1.* Multiplying or dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.

EXAMPLE

1/4 X 2/2 = 2/8 X 2/2 = 4/16. Then 1/4, 2/8, and 4/16 all represent the same fraction or value.

(2) *Rule 2.* Multiplying the numerator or dividing the denominator by a number multiplies the fraction by that number.

EXAMPLE

$$\frac{4 \times 3}{9} = \frac{12}{9} = \frac{4}{3} \qquad \frac{4}{9 \div 3} = \frac{4}{3}$$

(3) *Rule 3.* Dividing the numerator or multiplying the denominator by a number divides the fraction by that number.

EXAMPLE

$$\frac{4 \div 2}{7} = \frac{2}{7} \qquad \frac{4}{7 \times 2} = \frac{4}{14} = \frac{2}{7}$$

b. *Changing to a Given Denominator.* Changing a whole or mixed number, or a fraction, to a fraction of a given denominator:

(1) *Rule 1.* First change 1 to a fraction of the given denominator. Then multiply the numerator by the given whole number.

EXAMPLE

Reduce 5 to 6ths. Since 1 = 6/6, 5 = 5 x 6/6 = 30/6.

(2) *Rule 2.* With regard to a mixed number, change the whole number to a fraction. Then add to

the numerator of this fraction, the numerator of the fractional part of the mixed number.

EXAMPLE

Reduce 7 + 3/5 to 5ths.

1 = 5/5, 7 = 7 x 5/5 = 35/5

7 + 3/5 = 35/5 + 3/5 = 38/5

(3) *Rule 3*. To change a fraction to another fraction having a desired denominator, divide the desired denominator by the existing denominator. Then multiply both numerator and denominator of the existing fraction by the resulting quotient. In case of a mixed number, first change it to a fraction as described in the preceding rule, and then proceed as described.

EXAMPLE

Reduce 3/4 to 28ths.

28 + 4 = 7, 7 x 3 = 21, 3/4 = 21/28

c. *Changing an Improper Fraction to a Whole or Mixed Number*. Do the division shown in the following example. The quotient will be the number of units. If there is no remainder, it reduces to a whole number. If there is a remainder, it reduces to a mixed number of which the quotient is the whole number part and the remainder is the numerator of the fractional part.

EXAMPLE

Reduce 32/4 to a whole number.

32/4 = 32 + 4 = 8

Reduce 47/9 to a mixed number.

47/9 = 47 + 9 = 5 + 2/9

d. *Reducing a Fraction to its Lowest Terms*.

Rule 1. Divide both terms by a common factor or the greatest common divisor.

EXAMPLE

Reduce 75/105 to its lowest terms. Since dividing both the numerator and the denominator by the same number does not change the value of the fraction, both terms of the fraction may be divided by 5. Thus 75/105 = 15/21. Now both terms of 15/21 may be divided by 3; 15/21 = 5/7. Both 5 and 7 are prime to each other (no other number except 1 can be divided into both of then a whole number of times), so the fraction is now reduced to its lowest terms.

e. *Reducing Several Fractions having a Desired Common Denominator*.

Rule 1. Multiply both terms of each fraction by the quotient of the desired common denominator divided by the denominator of the fraction. Thus, the fraction 1/2 may be changed to 6ths by multiplying both its terms by a number which will make the denominator a 6th. This number is 3. Therefore, 1/2 becomes 3/6. Do likewise to change 1/3 to 6ths.

EXAMPLE

Reduce 1/2 and 1/3 to fractions which have 6 for a denominator. 1/2 = ?/6.

The first step is to divide 6 by 2. It goes 3 times. Therefore, you multiply the numerator by 3. You then get:

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \quad \text{Likewise} \quad \frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Reduce 7/9, 3/8, and 5/6 to 72ds. Both terms of 7/9 are multiplied by 8, since 72 ÷ 9 = 8. Both terms of 3/8 are multiplied by 9, since 72 ÷ 8 = 9. Both terms of 5/6 are multiplied by 12, since 72 ÷ 6 = 12. Therefore, 7/9 = 56/72, 3/8 = 27/72, and 5/6 = 60/72.

f. *Finding the Least Common Denominator of a Group of Fractions Having Different Denominators.*

Rule 1. To find the least common denominator of a group of fractions having different denominators, divide the given denominators by a prime number (a number divisible only by itself and 1) which will divide two or more of them. Similarly, divide the remaining numbers and their quotients. Continue this as long as possible. The LCD (least common denominator) will be the product of the divisors and the quotients of numbers left.

EXAMPLE

Find the LCD of 4/5, 5/6, 7/9, and 9/10.

Step 1. Divide the denominators 5 and 10 by the prime number 5. Write 1 under the 5 and 2 under the 10.

$$\begin{array}{r|rrrr} 5 & 5, & 6, & 9, & 10 \\ \hline & 1 & & & 2 \end{array}$$

Step 2. Bring down the denominators 6 and 9 as shown.

$$\begin{array}{r|rrrr} 5 & 5, & 6, & 9, & 10 \\ \hline & 1 & 6 & 9 & 2 \end{array}$$

Step 3. Divide the denominators 6 and 9 by the prime number 3. Bring down the other digits (remaining numbers), 1 and 2.

$$\begin{array}{r|rrrr} 5 & 5, & 6, & 9, & 10 \\ \hline 3 & 1, & 6, & 9, & 2 \\ \hline & 1, & 2, & 3, & 2 \end{array}$$

Step 4. Divide the digits (remaining numbers, 2 and 2, by the prime number 2. Bring down the digits (remaining numbers), 1 and 3. The remaining numbers 1 will not reduce any further as a whole number, and 3 is divisible by 3, but there is no second

number divisible by 3. No further division, therefore, is possible.

$$\begin{array}{r|l} 5 & 5,\ 6,\ 9,\ 10 \\ \hline 3 & 1,\ 6,\ 9,\ 2 \\ \hline 2 & 1,\ 2,\ 3,\ 2 \\ \hline & 1,\ 1,\ 3,\ 1 \end{array}$$

Step 5. Multiply the divisors (5, 3, and 2) and remaining quotients (1, 1, 3, and 1). The result is the LCD.

LCD = 5 x 3 x 2 x 1 x 1 x 3 x 1 = 90

The new fractions are 72/90, 75/90, 70/90, and 81/90.

6. Addition

a. *Finding the Sum of Two or More Fractions*.

Rule 1. To add together two or more fractions, first reduce them to fractions having an LCD (discussed in paragraph 5f). Then add the numerators. Next, write the sum of these numerators over the LCD.

EXAMPLE (see figure 6 on the following page)

Add 1/4 + 1/3.

Step 1: 12 is the LCD of 4 and 3

Step 2: 1/4 = 3/12 and 1/3 = 4/12 3/12 + 4/12 = 7/12

b. *Finding the Sum of Mixed Numbers*.

Rule 1. Find the LCD of the fractions. Add the whole numbers first. Then add the fractions. Add the sum of the fractions to the sum of the whole numbers. If the sum of the fractions is an improper fraction, reduce it to a mixed number.

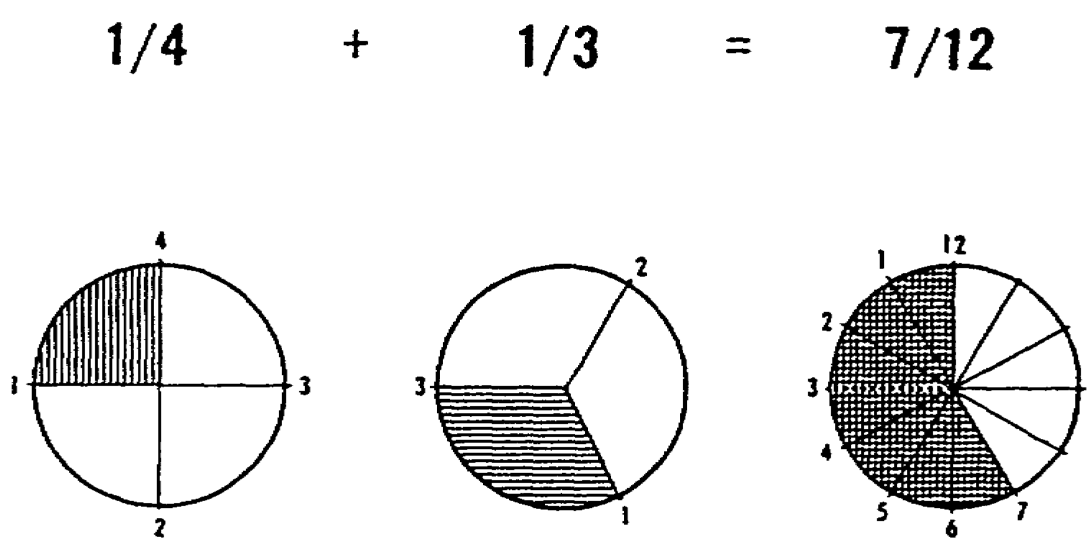
EXAMPLE

Add 20 2/3 + 13 1/2 + 7 1/8. The LCD is 24. Add the whole numbers first. Then add the fractions.

```
20 2/3 = 20 16/24
13 1/2 = 13 12/24
 7 1/8 =  7  3/24
         --------
         40 31/24
```

40 31/24 = 40 + 1 7/24 or 41 7/24.

FIGURE 6. ILLUSTRATING HOW 1/4 + 1/3 = 7/12.



7. Subtraction

a. *Finding the Difference*. In subtracting fractions, the difference between two fractions is found by taking one away from the other.

Rule 1. To subtract one fraction from another, make sure the fractions have a common denominator. If their denominators differ, reduce the fractions so that they have an LCD. Then subtract the numerators.

EXAMPLE

Find the difference between 5/6 and 8/15. The LCD is 30. Change both fractions to 30ths.

5/6 = 25/30

8/15 = 16/30

Subtract these fractions by subtracting their numerators, 25 - 16 = 9. Place the 9 over the denominator 30 and the result is 9/30 = 3/10.

b. *Subtracting Mixed Numbers*.

(1) *Rule 1*. To subtract mixed numbers, subtract the fractional and the whole parts separately. Then add the remainder of the fractions to the remainder of the whole numbers to get the answer in a mixed number.

EXAMPLE

From 27 5/6 take 14 5/8. Write it down as 27 5/6 - 14 1/8. The LCD of 8 and 6 is 24. The subtraction then reads:

27 5/6 = 27 20/24

-14 1/8 = -14 15/24

Subtract the numerators of the fractional part. Then subtract the whole numbers. Add these two results; the correct answer is 13 5/24.

(2) *Rule 2*. To subtract a fraction or mixed number from a whole number, such as 17 - 9/11, or to subtract a fraction from a mixed number in which the fraction of the minuend (the number from which another number is to be taken), is less than the fraction of the subtrahend (the number to be taken from the minuend), such as 12 5/8 - 7 7/8, borrow one from the whole number in the minuend. Add this to the fraction, making it an improper fraction. Then subtract.

EXAMPLE

From 12 5/8 take 7 7/8. 7/8 cannot be subtracted from 5/8, so borrow 8/8, or 1, from the 12; add this 8/8 to the 5/8, thus:

$$\begin{array}{rcrr} 12\ 5/8 & = & 11 & 13/8 \\ 7\ 7/8 & = & 7 & 7/8 \\ \hline & & 4 & 6/8 \text{ or } 4\ 3/4 \end{array}$$

8. Multiplication

a. *Finding the Product of a Fraction*. In multiplying fractions, they need not be reduced to an LCD, as in adding and subtracting fractions. When the numerator of a fraction is multiplied, the number of fractional units is multiplied. Their size (represented by the denominator) remains the same. But to multiply or increase the size of the fractional units (represented by the denominator), the denominator must be divided, and the number of fractional units (represented by the numerator) remains the same. Before multiplying, it is recommended that canceling of equal factors be carried out. (*Cancellation* is the process of striking out equal factors from the numerator and denominator of a fraction. This operation does not change the value of the fraction but aids in reducing it to its lowest terms.)

Rule 1. To multiply a fraction by an integer (a whole number), or an integer by a fraction, multiply the numerator by the integer. Reduce the product to its lowest terms. To multiply a fraction by a fraction, multiply the numerators together. This gives the numerator of the product. Next, multiply the denominators together. This gives the denominator of the product. Cancel where possible.

EXAMPLE

Multiply 3/5 by 4. This means find a fraction 4 times as great as 3/5.

$$\frac{3}{5} \times 4 = \frac{3 \times 4}{5} = \frac{12}{5} = 2\,\frac{2}{5}$$

Multiply 5/8 by 2/5..

$$\frac{5}{8} \times \frac{2}{5} = \frac{5 \times 2}{8 \times 5} = \frac{10}{40} = \frac{1}{4}$$

Time can be saved here by cancellation.

$$\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} = \frac{1}{4}$$

b. *Shop Work Problems*. In shop work, problems are met frequently that are condensed to written work looking something like this: 25 2/5 x 6 1/3, or 47 x 16 4/5.

Rule 1. To multiply two numbers, one or both of which are mixed numbers, first reduce the mixed numbers to improper fractions by multiplying the whole number by the denominator, and adding the numerator, as shown below. Then multiply, as in multiplying two fractions.

EXAMPLE

Multiply 2 2/7 by 5 1/4.

2 2/7 = 16/6

5 1/4 = 21/4

Change to improper fractions and cancel.

$$\frac{\overset{3}{\cancel{21}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{1}{\cancel{7}}} = \frac{12}{1} = 12$$

c. *Mixed Numbers*. When one number is a mixed number and the other a whole number, work as follows:

Multiply 7 3/5 x 6.

$$7\frac{3}{5} \times 6 = \frac{38}{5} \times \frac{6}{1} = \frac{228}{5} = 45\frac{3}{5}$$

9. Division

a. *General*. Division of fractions is the reverse of multiplication. For instance, dividing the numerator of a fraction reduces the number of fractional units, but the size of each unit remains the same. Multiplying the denominator reduces the size of the fractional units, but the number of fractional units remain the same. This is the same as inverting the divisor and multiplying. Thus, 7/8 ÷ 3/4 and 7/8 x 4/3 give the same answer. The fraction turned upside down is called the reciprocal.

b. *Dividing a Fraction by a Whole Number*.

Rule 1. Change the whole number to a fraction which has the number 1 as a denominator. Invert this fractional form. Then multiply.

EXAMPLE

Divide 7/11 by 3. Since 3 = 3/1, the reciprocal of 3 is then 1/3. 7/11 ÷ 3 = 7/11 x 1/3 = 7/33.

c. *Dividing a Whole Number by a Fraction*.

Rule 1. Invert the fraction and multiply.

EXAMPLE

Divide 13 by 3/7. Invert 3/7 to 7/3, then multiply. 13 ÷ 3/7 = 13 X 7/3 = 91/3 = 30 1/3.

d. *Dividing a Fraction by a Fraction*.

Rule 1. Invert the divisor, that is the second fraction, and multiply.

EXAMPLE

Divide 3/4 by 7/8. Invert 7/8 to 8/7, then multiply.

$$\frac{3}{4} \div \frac{7}{8} = \frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{7} = \frac{6}{7}$$

e. *Dividing a Mixed Number by a Fraction or by Another Mixed Number.*

Rule 1. Reduce the mixed numbers to improper fractions. Then invert the divisor and multiply, canceling where possible.

EXAMPLE

Divide 2 1/2 by 1 7/8.

$$\frac{5}{2} \div \frac{15}{8} = \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{15}}} = \frac{4}{3} = 1\ 1/3$$

10. Complex Fractions

Sometimes a necessity arises for solving a complex fraction; that is, one with a fraction in the numerator, or the denominator, or both, such as:

$$7/8 \text{ over } 4 \qquad \text{or} \qquad \frac{7/8}{4}$$

Rule 1. In solving the complex fraction, first reduce it to a simple fraction as shown below. The division by the second fraction is indicated. Then invert and multiply. Cancel when able.

EXAMPLE

$$\text{Divide } \frac{\frac{9}{11}}{\frac{2}{5}} \text{ by } \frac{3}{4}$$

$$9/11 \div 2/5 = 9/11 \times 5/2$$

$$9/11 \times 5/2 \div 3/4 = 9/11 \times 5/2 \times 4/3$$

$$\frac{\overset{3}{\cancel{9}}}{11} \times \frac{5}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}} = \frac{30}{11} = 2\,\frac{8}{11}$$

11. Conclusion

This task served to describe the processes for adding, subtracting, multiplying, and dividing fractions. It began with a review of the whole number system and the addition, subtraction, multiplication, and division of whole numbers. The next task will describe the processes for converting fractions to decimals and decimals to fractions; and adding, subtracting, multiplying, and dividing decimals.

LESSON 1

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF FRACTIONS AND DECIMALS; AND CONVERSION OF FRACTIONS TO DECIMALS AND DECIMALS TO FRACTIONS

TASK 2. Describe the processes for converting fractions to decimals and decimals to fractions; and for adding, subtracting, multiplying, and dividing decimals.

CONDITIONS

Within a self-study environment and given the subcourse text, without assistance.

STANDARDS

Within two hours

REFERENCES

No supplementary references are needed for this task.

1. Introduction

Task 1 served to discuss how to solve fractions in connection with machine shop operations. In task 2, the processes for converting fractions to decimals and decimals to fractions, and the addition, subtraction, multiplication, and division of decimals will be discussed.

2. General

Decimal fractions are simply common fractions written in a different form. The purpose of decimal fractions is to make work with fractions easier.

Many tools used in measuring very small dimensions are ruled off in decimals. In shop work, fractions of 8ths, 16ths, 32nds, and 64ths of an inch are used in making ordinary measurements. For greater accuracy in measurement, as for piston, bearing, and valve clearances or tolerances, and in machining some parts, decimals in the 100ths and 1,000ths of an inch are commonly used.

As was discussed in task 1, common fractions may be reduced to higher or to lower terms. Suppose that it becomes desirable to reduce all fractions to a standard denominator of 10, 100, 1,000, or even 100,000. Then 1/4 would become:

$$\frac{2\ 1/2}{10} \text{ or } \frac{25}{100} \text{ or } \frac{250}{1,000} \text{ or } \frac{2,500}{10,000}$$

These common fractions are now decimal fractions. But, instead of simplifying the work, they have complicated it with long, unhandy denominators. To relieve this complication, a symbol called a decimal point is used. This decimal point serves to eliminate the denominators. The problem fraction is then presented in its simplest form. For instance, 25/100 becomes .25. The decimal point has now taken the place of the denominator, which makes it easier to work with.

3. Definition of Terms

a. *Decimal Point*. The point (.) is called the decimal point. It is used to mark the beginning of the decimal fraction, or to separate it from a whole number.

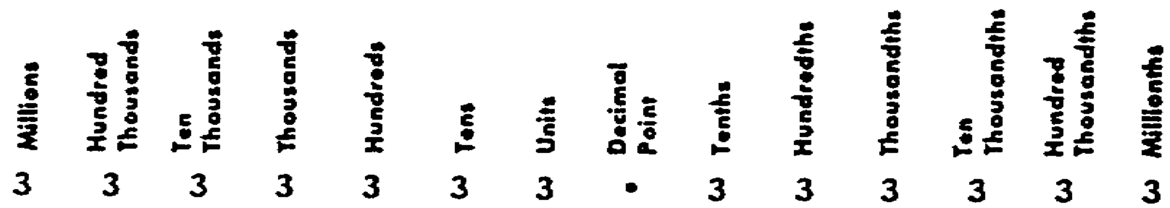
b. *Pure Decimal*. A decimal fraction containing only decimal places, such as .025, is a pure decimal. The example reads twenty-five thousandths.

c. *Mixed Decimal*. This is a whole number and a decimal fraction. For instance, 1.256 is a mixed decimal. It reads 1 and two hundred fifty-six thousandths.

4. Reading Decimals

Before attempting to read decimals, places should be learned. A study of figure 7 on the following page should help. The place in which you write a decimal point is very important. Each integer (a whole number) in figure 7 is a number 3. Yet, no two of these 3's have like values. The first number 3 to the left of the decimal point is a plain 3, meaning 3 whole units. The second 3 is three tens or thirty. In reading from the left of a decimal point, each unit increases in value by ten times the unit to its right.

FIGURE 7. DECIMAL PLACES.



What happens to the right of the decimal point? Figure 7 indicates that the first three to the right of the decimal point is three-tenths of one. The second three to the right of the decimal point is three-hundredths of one. In reading to the right of a decimal point, each unit decreases in value by one-tenth of the unit to its left. Thus, 333. is read three hundred thirty-three. If the decimal point is moved one place to the left, it becomes 33.3, or thirty-three and three-tenths. Again, move the decimal point to the left one space. It is now 3.33. This is read as three and thirty-three-hundredths.

5. Reduction of a Common Fraction to a Decimal Fraction

Rule 1. Add as many zeros to the numerator of the common fraction as you wish to have places in the decimal fraction. Then divide the resulting number by the denominator. Next, place the decimal point so as to make as many decimal places in the result as you have added zeros to the numerator.

EXAMPLES

Change 7/8 to its decimal equivalent.

$$8\overline{)7.000}$$ **Add zeros and divide**

0.875 Place decimal point

Change 2/7 to a decimal.

$$7\overline{)2.0000}$$

0.2857+

NOTE

In the second example, notice the (+) placed at the end of the decimal fraction. This means that the decimal fraction may be carried further if needed. A common fraction in its lowest terms can reduce to an exact decimal only when its denominator contains no prime factors other than 2 and 5. Thus, 3/64 reduces to an exact decimal, for 64 is made up of 2 x 2 x 2 x 2 x 2 x 2. On the other hand, 7/12 cannot be reduced to an exact decimal because its denominator contains a factor 3.

Table 1, on the following page, shows the decimal equivalents of the more common fractions.

6. Reduction of a Decimal Fraction to a Common Fraction

Rule 1. To form the denominator, replace the decimal point by a 1 followed by as many zeros as there are decimal places in the original fraction. Write in the figures to the right of the decimal point to form the numerator.

EXAMPLE

Change .5 to a common fraction. First change the decimal point to 10, which becomes the denominator; then write in the numerator, 5/10. Similarly, 2.75 becomes 2 75/100, which will reduce down to 2 3/4.

TABLE 1. DECIMAL EQUIVALENT OF SOME COMMON FRACTIONS.

Fraction	1/32nds	1/64ths	Decimal	Fraction	1/32nds	1/64ths	Decimal
		1	.015625			33	.515625
	1	2	.03125		17	34	.53125
		3	.046875			35	.546875
1/16	2	4	.0625	9/16	18	36	.5625
		5	.078125			37	.578125
	3	6	.09375		19	38	.59375
		7	.109375			39	.609375
1/8	4	8	.125	5/8	20	40	.625
		9	.140625			41	.640625
	5	10	.15625		21	42	.65625
		11	.171875			43	.671875
3/16	6	12	.1875	11/16	22	44	.6875
		13	.203125			45	.703125
	7	14	.21875		23	46	.71875
		15	.234375			47	.734375
1/4	8	16	.25	3/4	24	48	.75
		17	.265625			49	.765625
	9	18	.28125		25	50	.78125
		19	.296875			51	.796875
5/16	10	20	.3125	13/16	26	52	.8125
		21	.328125			53	.828125
	11	22	.34375		27	54	.84375
		23	.359375			55	.859375
3/8	12	24	.375	7/8	28	56	.875
		25	.390625			57	.890625
	13	26	.40625		29	58	.90625
		27	.421875			59	.921875
7/16	14	28	.4375	15/16	30	60	.9375
		29	.453125			61	.953125
	15	30	.46875		31	62	.96875
		31	.484375			63	.984375
1/2	16	32	.5	1	32	64	1.

7. Addition

Rule 1. First write the numbers so that the decimal points are directly under each other. Add as you do with whole numbers. Be sure to place the decimal point in the sum directly under the other decimal points.

EXAMPLE

Add the following:

36.036, 7.004, 0.00236, 427, 723.0026

Write them down as in ordinary addition. Watch the decimal points.

```
  36.036
   7.004
   0.00236
 427.
 723.0026
-----------
1193.04496
```

8. Subtraction

Rule 1. First write the numbers so that the decimal points fall under each other. Subtract as in whole numbers. Write the decimal point of the remainder directly under the other decimal points.

EXAMPLE

Subtract 46.8324 from 437.421. Write the number down as in ordinary subtraction. Be sure to place the decimal points directly under each other.

```
  437.421
-  46.8324
---------
  390.5886
```

9. Multiplication

Rule 1. To multiply decimal fractions, multiply as in whole numbers. Then count off from right to left as many decimal places in the product as there were in both factors, and place the decimal point in front of the last place counted off.

EXAMPLE

Multiply 7.32 by 0.032.

```
      7.32   Set down and
  x  0.032   multiply.
  ---------
      1464   Adding the decimal
     2196    places,
  ---------
   0.23424   you find 5.
```

Count off the 5 places from right to left in the product. Between this figure and the next one to the left, place the decimal point. You should now have as many figures to the right of the decimal point in the product as the total number you had in the two factors.

When a whole number or a decimal fraction is multiplied by 0.1, the decimal point is simply moved one place to the left. If multiplying by 0.01, the decimal point is moved two places to the left. The decimal point is moved three places left when multiplying by 0.001. If necessary, zeros may be added to the left of the multiplicand (the number to be multiplied). Thus, 32.4 x 0.0001 provides a product of 0.00324.

However, when a decimal fraction is multiplied by 10, move the decimal point one place to the right. When multiplying by 100, 1000, etc., simply move the decimal point to the right as many places as there are zeros in the multiplier.

10. Division

Rule 1. Set up the dividend and divisor as in division of whole numbers. Move the decimal point in the divisor to the right of the right-hand figure. Then move the decimal point in the dividend to the right, the same number of places that the point was moved in the divisor (add zeros to the dividend if necessary). Place the decimal point in the quotient directly above the new position of the decimal point in the dividend. Divide as in whole numbers. When dividing by 0.1, 0.01, 0.001, etc., move the decimal point one, two, three, etc., places in the dividend to the right, adding zeros if needed. Therefore, when dividing by 10,000 or 1,000 etc., move the decimal point one, two, three, etc., places in the dividend to the left.

EXAMPLE

Divide 0.4572 by 0.127.

```
            3.6
0.127. )0.457.2
         381
          762
          762
            0
```

Divide 4485 by .025.

```
          179400.
.025 )4485.000.
      25
      198
      175
       235
       225
        100
        100
          0
```

To divide 2.4 by 0.01, follow the rule above and move the decimal point in the dividend two places to the right. Add one zero, making the quotient 240.

$$2.4 \div 0.01 = 240$$

To divide 2.4 by 100, move the decimal point in the dividend two places to the left, adding one zero, making the quotient .024.

$$2.4 \div 100 = .024$$

11. Accuracy

In many cases, it is not practical or possible to carry out a problem to absolute accuracy. It may be practical to seek a result correct to a certain number of places of decimals only. If so, write a plus or minus sign (as the case may be) after the last figure to the right, such as .667- or .666+. The (+) sign is used to show that the result is actually larger than the one given. Likewise, the (-) sign is used to show that the result is actually less than the one given. In other cases, it may be advisable to round off a decimal to a given value as described in the rule below.

Rule 1. In writing the result of a calculation in decimal fractions to a certain number of places, write the last place as one figure larger if the next figure to the right is a 5 or larger. Should this figure be less than 5, discard it from the figure.

EXAMPLE

Suppose when solving a problem a result such as 52.56266666 is obtained. To write it correctly to three decimal places it is written as 52.563+. To write this same number in two decimal places, it is written 52.56+.

12. Conclusion

This task described the processes for converting fractions to decimals and decimals to fractions; and adding, subtracting, multiplying, and dividing decimals for the purpose of helping the machinist to achieve greater accuracy of measurements involved in his work. Now complete the practical exercise which is designed to reinforce your learning of the material presented in the two tasks of this lesson.

PRACTICAL EXERCISE 1

1. Instructions

Read the scenario and respond to the requirements that follow the scenario.

2. Scenario

You are the Repair Shop Technician in charge of the service section of a heavy maintenance company stationed in Texas. The shop officer has notified you that the maneuver elements of the division will participate in a field exercise scheduled to take place in 60 days. The maintenance company has been tasked to start mechanically preparing the maneuver element's equipment for this exercise. As a result of this tasking, the machine shop will have to fabricate a large number of repair parts and devices not found in the supply system, and still support the normal everyday mission requirements. The machine shop currently has its full authorization of two machinists. The new requirement, however, imposes an additional workload that cannot be completed in the allotted amount of time without additional machinists. There are two welders in your service section that are former machinists. Since the welding shop is currently not overloaded you decide to establish a second workshift by using these two welders as machinists to assist in completing the additional workload. However, you need to know the extent of their knowledge of machine shop calculations. You may have to provide them with refresher training. You have, therefore, developed a list of mathematical problems that will assist you in determining the extent of their knowledge of machine shop calculations.

3. Requirement

Below is the list of mathematical problems you developed. Prepare the answer sheet by solving these problems.

a. Which is the larger fraction 7/9 or 9/7?

b. Reduce 560/630 to its lowest terms.

c. Change the following to fractions having the least common denominator: 2/3, 3/4, 5/6, 7/8.

d. Add 14 3/4 + 30 1/2 + 4 and write the sum in the simplest form.

e. Subtract 6 2/9 from 12 17/90 and write the result in the simplest form.

f. Do the operations indicated and simplify: 7 3/4 - 4/5 + 2 17/20.

g. Multiply 21 x 2/3.

h. Divide 5/7 by 10.

i. Find the value of $\dfrac{3\ 2/7 \times 8\ 1/6}{4\ 2/3 \times 2\ 1/16}$

j. Change the decimal 00.00125 to a common fraction, and reduce it to its lowest term.

k. Add: 2.367 + 45.002 + 0.401 + 7.64.

l. Subtract: 75.7575 - 55 1/8.

m. Multiply: 2.53 x 0.00635.

n. Divide: 43.769 ÷ 4.76 and show the results to four decimal places.

LESSON 1. PRACTICAL EXERCISE - ANSWERS

1. Requirement

a. 9/7

b. 8/9

c. 16/24, 18/24, 20/24, 21/24

d. 49 1/4

e. 5 29/30

f. 9 4/5

g. 14

h. 1/14

i. 2 26/33

j. 1/800

k. 55.410

l. 20.6325

m. 0.0160655

n. 9.1952

LESSON 2

CONVERSION OF LINEAR MEASUREMENTS FROM THE ENGLISH TO THE METRIC SYSTEM AND FROM THE METRIC TO THE ENGLISH SYSTEM; AND SOLVING PROBLEMS USING RATIO, PROPORTION, AND TRIGONOMETRY

TASK 1. Describe the processes for converting linear measurements from the English to the metric system and from the metric to the English system.

CONDITIONS

Within a self-study environment and given the subcourse text, without assistance.

STANDARDS

Within two hours

REFERENCES

No supplementary references are needed for this task.

1. Introduction

Lesson 1 provided a review of the whole number system and discussed the processes for solving machine shop work problems through addition, subtraction, multiplication, and division of fractions and decimals. It also discussed the conversion of decimals to fractions and fractions to decimals. To provide a complete coverage of machine shop calculations, Lesson 2 will discuss the processes for converting linear measurements between the English and metric systems, and for solving machine shop problems involving the use of ratio, proportion, and trigonometry. This task will focus on the conversion of linear measurements from the English to the metric system and from the metric to the English system.

2. Linear Measure

a. *General*. Linear measure is the measurement of line distance. In machine shop calculations, it is important to know how to convert linear measurements from the English to the metric system and vice-versa. Subsequent paragraphs provide an explanation of both these systems and how to convert from one system to another.

b. *English System*. This system consists, basically, of the inch, foot, yard, and mile. The foot is the basic unit of measure. The inch is a subdivision of the foot, while the yard and the mile are multiples of the foot. Table 2 below depicts the English system.

TABLE 2. ENGLISH SYSTEM LINEAR MEASUREMENTS.

12 inches (in)	=	1 foot (ft)
3 feet (ft)	=	1 yard (yd)
5280 feet (ft)	=	1 mile
1760 yards (yd)	=	1 mile

c. *Metric System*. This system is based on the decimal system, just like the United States dollar (10 cents equals a dime, and 10 dimes equal one dollar). The meter is the basic unit of measurement, as depicted in Table 3 below. As shown in this table, units that are multiples or fractional parts of the meter, such as the millimeter, are designated as such by prefixes to the word meter.

TABLE 3. METRIC SYSTEM LINEAR MEASUREMENTS.

10 millimeters (mm)	=	1 centimeter (cm)
10 centimeters (cm)	=	1 decimeter (dm)
10 decimeters (dc)	=	1 meter (m)
1000 meters (m)	=	1 kilometer (km)

3. Conversion

a. *Requirements for Conversion*. To convert from one system to another, a knowledge of equivalent values between these two systems is necessary. Table 4 provides a list of equivalent values between the two systems for linear measurements equal to or greater than one inch. Table 5 on the following page provides the metric equivalents of linear measurements of less than an inch.

TABLE 4. LINEAR MEASUREMENTS EQUIVALENT VALUES.

1 mil = 0.001 in
1 millimeter = 39.370 mils
1 millimeter = 0.039370 in
1 millimeter = 0.001 in
1 centimeter = 0.3937 in
1 centimeter = 0.0328 ft
1 centimeter = 0.01 m
1 meter = 39.37 in
1 inch (in) = 1000 mils
1 inch = 25.440 mm
1 inch = 2.540 cm
1 inch = 0.0833 ft or 1/12
1 inch = 0.02777 yd or 1/36
1 inch = 0.0254 m, aprx. 1/40
1 foot (ft)(U.S.) = 304.801 mm
1 foot = 30.480 cm
1 foot = 12 in
1 foot = 0.333 yd or 1/3
1 foot = 0.3048 m, aprx. 3/10
1 foot = 0.000304 km
1 foot = 0.000189 mile
1 yard (yd)(U.S.) = 91.440 cm
1 yard = 36 in
1 yard = 3 ft
1 yard = 0.914 m
1 yard = 0.000914 km
1 yard = 0.000568 mile
1 mile = 5280 ft
1 mile = 1760 yd
1 mile = 1609.35 m
1 mile = 1.609 km
1 mile = 0.868 nautical mile
1 kilometer = .62 mile

TABLE 5. DECIMAL AND METRIC EQUIVALENTS
OF FRACTIONS OF AN INCH.

Fraction	1/32nds	1/64ths	Decimal	Millimeters	Fraction	1/32nds	1/64ths	Decimal	Millimeters
		1	.015625	0.3968			33	.515625	13.0966
	1	2	.03125	0.7937		17	34	.53125	13.4934
		3	.046875	1.1906			35	.546875	13.8903
1/16	2	4	.0625	1.5875	9/16	18	36	.5625	14.2872
		5	.078125	1.9843			37	.578125	14.6841
	3	6	.09375	2.3812		19	38	.59375	15.0809
		7	.109375	2.7780			39	.609375	15.4778
1/8	4	8	.125	3.1749	5/8	20	40	.625	15.8747
		9	.140625	3.5718			41	.640625	16.2715
	5	10	.15625	3.9686		21	42	.65625	16.6684
		11	.171875	4.3655			43	.671875	17.0653
3/16	6	12	.1875	4.7624	11/16	22	44	.6875	17.4621
		13	.203125	5.1592			45	.703125	17.8590
	7	14	.21875	5.5561		23	46	.71875	18.2559
		15	.234375	5.9530			47	.734375	18.6527
1/4	8	16	.25	6.3498	3/4	24	48	.75	19.0496
		17	.265625	6.7467			49	.765625	19.4465
	9	18	.28125	7.1436		25	50	.78125	19.8433
		19	.296875	7.5404			51	.796875	20.2402
5/16	10	20	.3125	7.9373	13/16	26	52	.8125	20.6371
		21	.328125	8.3342			53	.828125	21.0339
	11	22	.34375	8.7310		27	54	.84375	21.4308
		23	.359375	9.1279			55	.859375	21.8277
3/8	12	24	.375	9.5248	7/8	28	56	.875	22.2245
		25	.390625	9.9216			57	.890625	22.6214
	13	26	.40625	10.3185		29	58	.90625	23.0183
		27	.421875	10.7154			59	.921875	23.4151
7/16	14	28	.4375	11.1122	15/16	30	60	.9375	23.8120
		29	.453125	11.5091			61	.953125	24.2089
	15	30	.46875	11.9060		31	62	.96875	24.6057
		31	.484375	12.3029			63	.984375	25.0026
1/2	16	32	.5	12.6997	1	32	64	1.	25.3995

b. *Converting From the English System to the Metric System.*

(1) *Rule 1*. To convert miles to kilometers, multiply by 1.61.

EXAMPLE

Reduce 12 miles to kilometers.

12 x 1.61 = 19.32 km

(2) *Rule 2*. To convert yards to meters, multiply by 0.9144.

EXAMPLE

Reduce 3 yards to meters.

3 x 0.9144 = 2.7432 m

(3) *Rule 3*. To convert inches to centimeters, multiply by 2.54.

EXAMPLE

Reduce 16 inches to centimeters.

16 x 2.54 = 40.64 cm

(4) *Rule 4*. To convert inches to millimeters, multiply by 25.4.

EXAMPLE

Reduce 2 feet 8 inches to millimeters.

2 ft = 24 in + 8 in = 32 in

32 in x 25.4 = 812.8 mm

(5) *Rule 5*. To convert inches to millimeters multiply by 25.4.

EXAMPLE

Reduce 11 inches to millimeters.

11 x 25.4 = 279.4 mm

c. *Converting from the Metric System to the English System*.

(1) *Rule 1*. To convert kilometers to miles, multiply by 0.62.

EXAMPLE

Reduce 60 kilometers to miles.

60 x 0.62 = 37.2 miles

(2) *Rule 2*. To convert meters to yards multiply by 1.0936.

EXAMPLE

Reduce 3.5 meters to yards.

3.5 x 1.0936 = 3.8276 yd

(3) *Rule 3*. To convert meters to inches multiply by 39.37.

EXAMPLE

Reduce 1.6 meters to inches.

1.6 x 39.37 = 62.99 in

(4) *Rule 4*. To convert centimeters to inches multiply by 0.3937.

EXAMPLE

Reduce 76.2 centimeters to inches.

76.2 x 0.3937 = 30 in

(5) *Rule 5*. To convert millimeters to inches multiply by 0.03937.

EXAMPLE

Reduce 88.9 millimeters to inches.

88.9 x 0.03937 = 3.5 in

d. Metric equivalents of less than an inch are expressed in terms of millimeters. To convert linear measurements of less than an inch to millimeters and millimeter linear measurements to fractions of an inch use Table 5 on page 37 for the more common fractions. Otherwise, follow the procedure stated above and in paragraph 5 (Reduction of a common fraction to a decimal fraction), beginning on page 24, and paragraph 6 (Reduction of a decimal fraction to a common fraction), beginning on page 25.

LESSON 2

CONVERSION OF LINEAR MEASUREMENTS FROM THE ENGLISH TO THE METRIC SYSTEM AND FROM THE METRIC TO THE ENGLISH SYSTEM; AND SOLVING PROBLEMS USING RATIO, PROPORTION, AND TRIGONOMETRY

TASK 2. Describe the processes for solving problems using ratio and proportion.

CONDITIONS

Within a self-study environment and given the subcourse text, without assistance.

STANDARDS

Within two hours

REFERENCES

No supplementary references are needed for this task.

1. Introduction

A major part of machine shop work involves the fabrication of such parts as a spool, gear, or pulley for machinery and vehicle powertrains. These type parts must be machined to a predetermined size that will enable their turning at a given number of revolutions per minute (rpm). The machining of these parts requires knowledge of the mathematical processes involved in determining the size to which these items must be machined. This task, therefore, is designed to provide the processes for determining the size of these parts through the use of mathematical problems involving ratio and proportion. Subsequent paragraphs provide an explanation of the methods for solving ratio and proportion problems.

2. Ratio and Proportion

a. *General.* Ratio and proportion are methods for reducing the confusion and minimizing the possibilities of error in working arithmetic problems. A working knowledge of these methods makes it easier to solve many shop problems. The language of ratio and proportion is mostly a sign language. Letters and symbols are commonly used in place of long numbers and represent unknown quantities and values.

b. *Ratio.*

(1) Ratio is the relation which one quantity bears to another quantity of the same kind. It is used extensively in shop work. Shop drawings or blueprints are generally drawn to scale. Scale means one figure is used to represent another. Usually a small figure represents a larger figure. For example, on a blueprint 1 inch might represent 1 foot.

(2) The two numbers used in the ratio are called the "terms" of the ratio. The first number of a ratio is called the antecedent; the second number is called the consequent. The consequent is the divisor. The colon (:) is the sign of ratio and means "is to." Thus, 3 : 5 reads "3 is to 5." It is in effect a dividing sign without the dividing (-) line. Such other expressions as "in the same ratio", "in the same proportion', or "pro rata" all have the same meaning.

(3) The ratio of one number to another is really the quotient of the first number divided by the second number.

EXAMPLE

Determine the ratio of the expression 8 : 2.

$8 : 2 = 4$	Divide 8 by 2. Thus, the ratio or value of 8 to 2 is 4.

(4) The value of a ratio is not changed by either multiplying or dividing both terms by the same number.

EXAMPLE

Multiply the expression 3 : 2 by 2.

3 : 2 = 6 : 4	Multiplying both terms by 2 renders the expression 6 : 4. Check by dividing 3 by 2 which renders a ratio of 1.5. In the second expression dividing 6 by 4 also renders a ratio of 1.5. Thus, 3 : 2 is equal in value to 6 : 4.

Divide the expression 8 : 4 by 4.

8 : 4 = 2 : 1	Dividing both terms by 4 renders the expression 2 : 1. Check by dividing 8 by 4 in the first expression, which renders a ratio of 2. In the second expression dividing 2 by 1 renders a ratio of 2. Thus 8 : 4 is equal in value to 2 : 1.

c. *Proportion*.

(1) Proportion is a statement of equality between two ratios. Thus, 3 : 4 : : 6 : 8. The symbol (: :) means "as" or "equals." Either this symbol or the equal sign (=) may be used. The "extremes' are the first and last terms. The "means" are the second and third terms.

(2) *Rule 1*. In proportion, the product of the means equals the product of the extremes.

EXAMPLE

Therefore, 3 : 4 : : 9 : 12.

4 x 9 = 36	Multiply the means.
3 x 12 = 36	Multiply the extremes. Thus, the two expressions are equal.

NOTE: This makes it possible to find an unknown quantity. In other words, when three terms of a proportion are known the fourth can be found.

(3) *Rule 2. To find one unknown mean*. When both extremes and one mean are known, the unknown mean can be found by dividing the product of the extremes by the known mean.

EXAMPLE

Find the unknown mean for 15 : 5 = ? : 20.

$15 : 5 = X : 20$ $15 \times 20 = 5 \times X$ $300 = 5X$ $60 = X$	Let "X" represent the unknown mean. Multiply the extremes and means. Divide both sides of the equation by 5. The unknown mean is 60.

(4) *Rule* 3. *To find one unknown extreme*. When both means and one extreme are known, find the unknown extreme by dividing the product of the means by the known extreme.

EXAMPLE

Find the unknown extreme for ? : 28 :: 2 : 8

$X : 28 :: 2 : 8$ $X = \dfrac{28 \times 2}{8} = 7$	Let "X" represent the unknown extreme. Multiply the means. Divide by the known extreme. The unknown extreme is 7.

d. *Inverse Proportion*.

(1) The ratio 2 : 3 is the inverse of the ratio 3 : 2. In proportion, when a ratio is equal to its inverse, the elements are said to be inversely proportional.

(2) Two numbers are inversely proportional when one increases as the other decreases. In this case their product is always the same. A practical example of inverse ratio is seen in problems dealing with pulleys.

(a) *Rule 1.* The speed of pulleys connected by belts are inversely proportional to their diameters. The smaller pulley rotates faster then the larger pulley.

EXAMPLE

A 24 inch pulley fixed to a live shaft which makes 400 revolutions per minute (rpm) is belted to a 6 inch pulley, as shown in figure 8 on the following page. Find the rpm of the smaller pulley.

This is what the problem looks like:

A is the driving pulley, B is the driven pulley.
Then, X : 400 :: 24 : 6

$$X = \frac{400 \times \overset{4}{\cancel{24}}}{\underset{1}{\cancel{6}}} = 1600 \text{ rpm}$$

(b) *Rule 2.* The speed of gears running together is inversely proportional to their number of teeth.

EXAMPLE

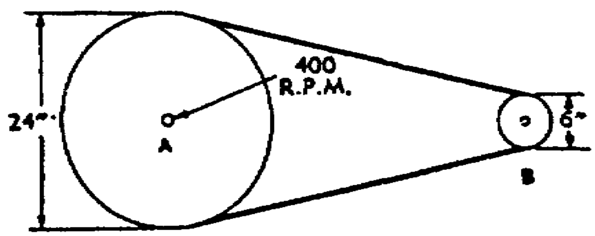
A driving gear with 48 teeth meshes with a driven gear which has 16 teeth. If the driving gear makes 100 rpm, find the number of rpm of the driven gear.

$$X = \frac{100 \times \overset{3}{\cancel{48}}}{\underset{1}{\cancel{16}}} = 300 \text{ rpm}$$

3. Pulley Trains and Gear Trains

a. In the previous paragraph, we discussed the meanings and methods of solving ratio and proportion problems. In this paragraph, we will apply these methods to help determine the size to which a pulley or gear should be machined in order to enable it to rotate at a given number of revolutions per minute (rpm) for efficient operation of the machinery pulley train or vehicle gear train. A pulley train is a series of pulleys connected by belting as shown in figure 9 on page 47. A gear train is a series of gears running

FIGURE 8. PULLEYS.



together; the power comes from one of the pulleys or gears. Neglecting slippage of the belting in a pulley train, the same method of determining relative sizes applies to both systems.

EXAMPLE

Find the rpm of the 6 inch pulley shown in figure 9 on the following page.

RPM of pulley (C)		RPM of pulley (A)		Diameters of pulleys (A) and (B)		Diameters of pulleys (C) and (B)
X	:	200	=	(15 x 12)	:	(10 x 6)

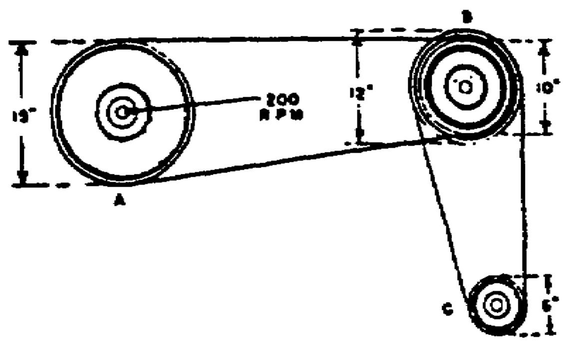
Therefore

$$\frac{\overset{20}{\cancel{200}} \times 15 \times \overset{2}{\cancel{12}}}{\underset{1}{\cancel{10}} \times \underset{1}{\cancel{6}}} = 600 \text{ rpm}$$

b. *Screw Gearing.*

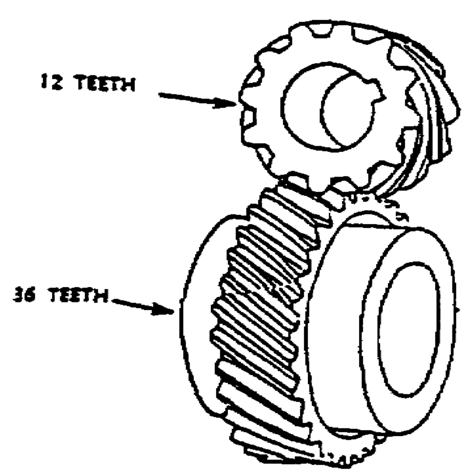
(1) *Spiral.* Gears are often used to reduce speed. The teeth on the gears are arranged in the same manner as the threads of a screw. A spiral gear may have any number of teeth. A one-toothed gear corresponds to a single-threaded screw. A

FIGURE 9. PULLEY TRAIN.



many-toothed gear corresponds to a many-threaded screw. Look at figure 10. Count the teeth in the upper gear. There are 12. This gear, then, equals a 12-threaded screw. The lower gear has 36 teeth. It corresponds to a 36-threaded screw. Hence, the small gear makes 3 complete turns while the large gear is making 1.

FIGURE 10. SPIRAL GEAR SYSTEM.



(2) *Worm and Worm Gearing*. Worm gearing is used to transmit power between two shafts at 90° to each other, but not in the same plane. In worm gearing, the velocity ratio is the ratio between the number of teeth on the gears and the number of threads on the worm. Figure 11 shows a worm and a single-threaded worm gear.

EXAMPLE

Find the revolutions per minute (rpm) for the worm gear.

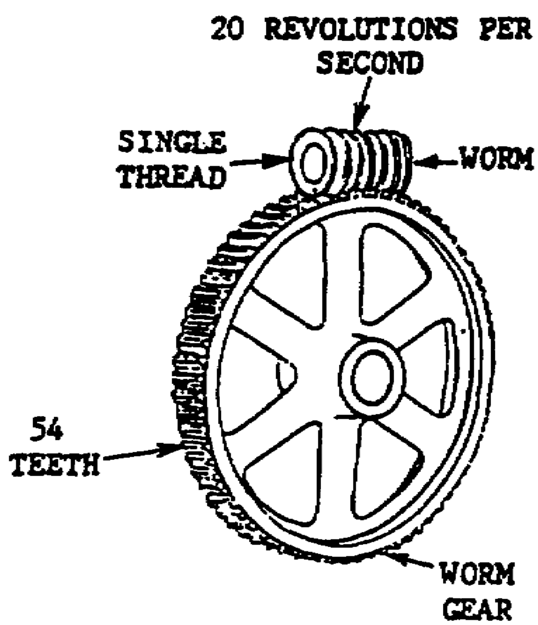
$20 \times 60 = 1200$	Convert 20 revolutions-per-second of the single worm to rpm by multiplying by 60 seconds.

Therefore

$$X = \frac{\overset{600}{\cancel{1200}} \times 1}{\underset{27}{\cancel{54}}}$$

$$X = 22.22 \text{ or } 22\ 2/9 \text{ rpm}$$

FIGURE 11. WORM GEAR SYSTEM.



LESSON 2

CONVERSION OF LINEAR MEASUREMENTS FROM THE ENGLISH TO THE METRIC SYSTEM AND FROM THE METRIC TO THE ENGLISH SYSTEM; AND SOLVING PROBLEMS USING RATIO, PROPORTION, AND TRIGONOMETRY

TASK 3. Describe the processes for solving problems using trigonometry.

CONDITIONS

Within a self-study environment and given the subcourse text, without assistance.

STANDARDS

Within two hours

REFERENCES

No supplementary references are needed for this task.

1. Introduction

In this task we will discuss the processes involved in solving machine shop problems through the use of trigonometry.

2. General

Task 1 and 2 of this lesson served respectively to describe the processes for converting linear measurements from the English to the metric system and from the metric to the English system, and for solving problems using ratio and proportion. In view of the fact that some military equipment has been developed with both metric and English measured components, Task 1 enables the machinist to convert linear measurements from one system to another, thereby ensuring the proper mating of machined parts that must operate in mesh with each other. Task 2 enables the machinist to determine the size that a spool, pulley or gear must be machined to, to permit its rotation at a specified rpm in the pulley or gear train of military machinery and vehicles.

Task 3 will describe the processes for solving problems through triangulation, otherwise known as trigonometry. Trigonometry is essentially that branch of mathematics which deals with the relations existing between the sides and angles of triangles. In this task only right triangles will be discussed. A right triangle is a triangle that contains one 90° angle and two other lesser angles for a total of 180° or half the number of degrees in a circle, which contains 360°.

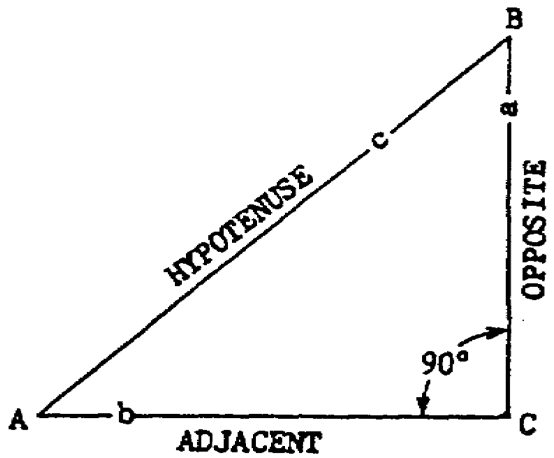
This process will assist the machinist in determining the pitch or angle of screw threads, gear teeth, and tapers for parts that must be fabricated for items not normally available through supply channels, or in an emergency in a combat situation. Before going into the solving of trigonometric problems, let's first review the trigonometric functions which are the basis for solving these types of problems.

3. Trigonometric Functions

a. For any given acute angle in a right triangle, certain ratios exist among the sides. These ratios are called "trigonometric functions." They determine sides and angles in a right triangle. To this end, the sides of a right triangle are given certain names to indicate their relation to the angles. Thus, in any right triangle, such as shown in figure 12 on the following page, the side "c," which is opposite to the right angle "C," is called the "hypotenuse"; side "a" is opposite angle "A" and is called the "opposite side"; side "b," is adjacent to angle "A" and is called the "adjacent side." Notice, however, that when the sides refer to angle "B," side "b" is the opposite side and side "a" is the adjacent side. However, the hypotenuse, the longest side, is always called the hypotenuse with reference to either angle.

b. In this triangle, it is possible to show six different ratios of the sides. They are a/c, b/c, a/b, b/a, c/b, and c/a. An explanation of these ratios follows using the ratio a/c as an example. This explanation is also applicable to the other ratios; a/c means the same as "a" divided by "c,"

FIGURE 12. SIDES IN REFERENCE TO ANGLE "A."



or "a" over "c." In each ratio the letter in the same position as "a" in relation to "c" represents the numerator, which can be given a numerical value. The letter "c" in relation to the position of "a" represents the denominator, which can be given a numerical value. If the letter "a" is assigned the numerical value of 2 and the letter "c" is assigned the numerical value of 4, the mathematical expression would be that the number 2 must be divided by the number 4. Thus, 2/4 = .5. These ratios are the trigonometric functions as described below:

(1) The ratio a/c, is called the sine A and is written sin A. **or** $$\frac{\textbf{opposite side}}{\textbf{hypotenuse}}$$

(2) The ratio b/c, is called the cosine A and is written cos A. **or** $$\frac{\textbf{adjacent side}}{\textbf{hypotenuse}}$$

(3) The ratio a/b is called the tangent A and is written tan A. **or** $$\frac{\textbf{opposite side}}{\textbf{adjacent side}}$$

(4) The ratio b/a is called the cotangent A and is written cot A. **or** $$\frac{\textbf{adjacent side}}{\textbf{opposite side}}$$

(5) The ratio c/b is called the secant A and is written sec A. or

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

(6) The ratio c/a is called the cosecant A and is written csc A. or

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite side}}$$

c. The trigonometric functions discussed here will be limited to the sine, cosine, tangent, and cotangent, since practically every common shop problem in trigonometry can be solved by means of these functions. The values of the trigonometric functions in terms of the names of the sides should be learned. To assist in learning these functions, use the example below.

EXAMPLE

Using the lettered and named sides of the triangle (figure 12 on the previous page), write the ratios for sin A, cos A, tan A, cot A, sin B, cos B, tan B, and cot B.

sin A = a/c; cos A = b/c; sin B = b/c;
cos B = a/c; and tan A = a/b; cot A = b/a;
tan B = b/a; cot B = a/b.

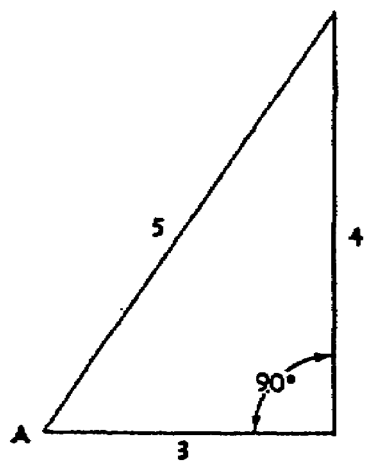
Therefore sin A = cos B and cos A = sin B
tan A = cot B and cot A = tan B

d. A trigonometric function expresses the value of an angle in terms of the sides of the right triangle containing that angle. For instance, the value of angle A in figure 13 on the following page may be expressed as:

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Thus, if the function and the dimension of one of the sides of that function ratio are known, then the dimension of the other side can be found.

FIGURE 13. VALUE OF AN ANGLE IN TERMS OF THE SIDES.



e. The rules for identifying angles and sides of right triangles are:

(1) $$\text{Sine} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

(2) $$\text{Cosine} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

(3) $$\text{Tangent} = \frac{\text{side opposite}}{\text{side adjacent}}$$

(4) $$\text{Cotangent} = \frac{\text{side adjacent}}{\text{side opposite}}$$

(5) Side opposite = hypotenuse x sine

(6) Side opposite = side adjacent x tangent

(7) Side opposite = side adjacent ÷ cotangent

(8) Side adjacent = hypotenuse x cosine

(9) Side adjacent = side opposite x cotangent

(10) Side adjacent = side opposite ÷ tangent

(11) Hypotenuse = side opposite ÷ sine

(12) Hypotenuse = side adjacent ÷ cosine

f. Procedure for using these Rules.

(1) In a right triangle, both the known and unknown sides (opposite, adjacent, and hypotenuse) of the problem are named.

(2) Choose from among the previous rules; select one that fits the given numerical values.

(3) Substitute the given values in the rule and solve for the unknown.

EXAMPLE

Find side “a” if sin A = 3/5 and side “c” = 20.5 (figure 14 on the following page). Here the sine of angle A is given, and “a” is the side opposite. According to rule (5) in paragraph 3e on page 53, side opposite = hypotenuse x sine. Substituting 20.5 for hypotenuse and 3/5 for sine, we get: side opposite = 20.5 x 3/5 = 12.3.

Find “b” if cos A = .44 and “c” = 3.5 (figure 15 on the following page). Here the cosine of angle A is given, and “b’ is the side adjacent. According to rule (8) in paragraph 3e on page 53, side adjacent = hypotenuse x cosine. Substituting 3.5 for hypotenuse and .44 for cosine, the side adjacent = 3.5 x .44 = 1.54.

Find “a” if tan A = 11/3 and “b” = 2 5/11 (figure 16 on page 56). Here the tangent of angle A is given, and “a” is the side opposite. According to rule (6) in paragraph 3e on page 53, side opposite = side adjacent x tangent. Substituting 2 5/11 for side adjacent and 11/3 for tangent, side opposite = 2 5/11 x 11/3 = 9.

Find “b” if cot A = 4 and “a” = 17 (figure 17 on page 56). Here the cotangent of angle A is given, and “b” is the side adjacent. According to rule (9) in paragraph 3e on page 53, side adjacent = side opposite x cotangent. Substituting 17 for side opposite and 4 for cotangent, side adjacent = 17 x 4 = 68.

FIGURE 14. FIND THE SIDE OPPOSITE ANGLE "A."

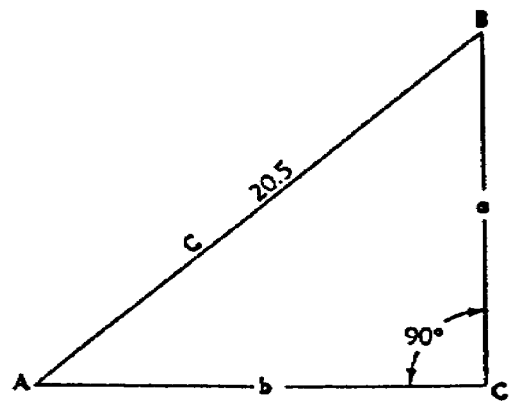


FIGURE 15. FIND SIDE ADJACENT TO ANGLE "A."

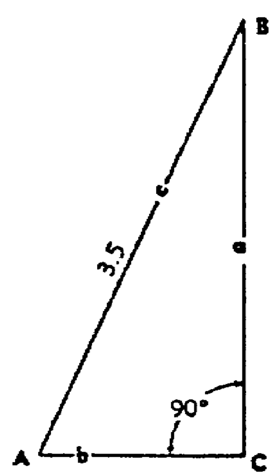


FIGURE 16. FIND SIDE OPPOSITE ANGLE "A."

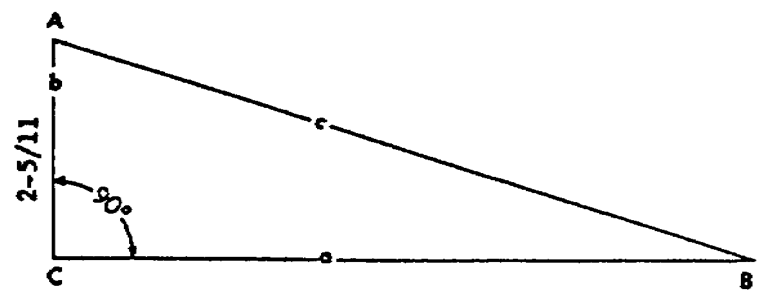
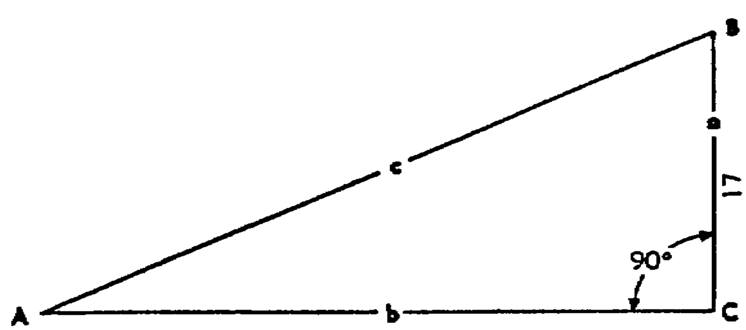


FIGURE 17. COTANGENT GIVEN, FIND ADJACENT SIDE.



4. Calculations with Angles

a. To add angles, arrange the degrees, minutes, and seconds in separate columns and add each column separately. Remember, in angles 60 seconds makes a minute and 60 minutes makes a degree. Therefore, if the "seconds" column adds up to 60 or more, subtract 60, or a multiple of 60, from that column and add one minute, or the same multiple of one minute, to the minutes column. If the "minutes" column adds up to 60 or more, proceed similarly, remembering that seconds change to minutes and minutes to degrees.

EXAMPLE

```
20°   40'    25"
 8°   35'     5"
30°   58'    51"
----------------
58°  133'    81"
       +1   -60"
----------------
58°  134'    21"
+2  -120'
----------------
60°   14'    21"
```

Take away 60″ from 81″ and add 1′ to 133′. Then take away 120′ from 134′ and add 2° to 58°. After becoming accustomed to this procedure, the values can be carried to their respective columns mentally.

b. To subtract angles, arrange the degrees, minutes, and seconds in separate columns, with the larger angle on top. Then subtract the individual columns. If the upper number in a column is too small to allow subtraction, one unit must be taken away from the following column and 60 units added to the insufficient number. This makes the subtraction possible.

EXAMPLE

Subtract 14° 51′ 30″ from 86° 45′ 10″.

```
86°   45'    10"
14°   51'    30"
-----------------
85°  104'    70"
14°   51     30
-----------------
71°   53     40"
```

Here the subtraction cannot be performed in either the seconds or the minutes columns. Hence take away 1′ from 45′ leaving 44′, and add 60″ to 10″, getting 70″. Also, take 1° from 86°, leaving 85°, and add 60′ to 44′, getting 104′.

c. To multiply an angle by a number, it is necessary to multiply each column by the given number. Then, if the answers in the "seconds" or the "minutes" columns are greater then 60, they must be reduced, as in the addition of angles.

EXAMPLE

Multiply 15° 21′ 40″ by 3.

```
15°   21'   40"
              x 3
------------------
45    63'  120"  =  45°  65'  0" = 46° 5'
```

d. To divide an angle by a given number, it is necessary to divide each column by the number, beginning with the "degrees" column. The remainder in degrees, if any, must be changed into minutes and added to the "minutes" column. The division is then performed on the "minutes" column. The remainder in minutes, if any, must be changed into seconds and added to the "seconds" column. Finally, the division is performed on the "seconds" column.

EXAMPLE

Divide 71° 22′ 42″ by 3.

```
               2° X 60 = 120'         1' X 60 = 60"
                         + 22'                  +42"
                          ---                   ----
3  71°                    142                   102"
---------------------------------------------------
   23°                    47'                    34"
---------------------------------------------------
    2°                     1'
```

Answer: 23° 47′ 34″

5. Trigonometric Tables

a. *General*. In order to facilitate the solution of trigonometric problems, tables have been prepared which give numerical values to the sine, cosine, tangent, and cotangent of angles from 0° to 90°.

b. *Use of Tables*.

(1) Tables 6 and 7, on the following pages, are an excerpt from the trigonometric tables listed in Appendix C-4 of FM 43-3. Notice that the heading for degrees (that is, 43, 44, 45, and 46) appear both at the top and at the bottom of each page.

The “minute” column at the left of each page is read from the top toward the bottom, while the “minute” column at the right is read from the bottom toward the top. Notice, also, that the functions at the top and bottom of each column are different. The values of the functions are given to five places of decimals.

TABLE 6. TRIGONOMETRIC TABLES.

SINES AND COSINES, NATURAL — TANGENTS AND COTANGENTS, NATURAL

	43°		44°		43°		44°		
	Sine	Cosin	Sine	Cosin	Tang	Cotang	Tang	Cotang	
0	.68200	.73135	.69466	.71934	.93252	1.07237	.96569	1.03553	60
1	.68221	.73116	.69487	.71914	.93306	1.07174	.96625	1.03493	59
2	.68242	.73096	.69508	.71894	.93360	1.07112	.96681	1.03433	58
3	.68264	.73076	.69529	.71873	.93415	1.07049	.96738	1.03372	57
4	.68285	.73056	.69549	.71853	.93469	1.06987	.96794	1.03312	56
5	.68306	.73036	.69570	.71833	.93524	1.06925	.96850	1.03252	55
6	.68327	.73016	.69591	.71813	.93578	1.06862	.96907	1.03192	54
7	.68349	.72996	.69612	.71792	.93633	1.06800	.96963	1.03132	53
8	.68370	.72976	.69633	.71772	.93688	1.06738	.97020	1.03072	52
9	.68391	.72957	.69654	.71752	.93742	1.06676	.97076	1.03012	51
10	.68412	.72937	.69675	.71732	.93797	1.06613	.97133	1.02952	50
11	.68434	.72917	.69696	.71711	.93852	1.06551	.97189	1.02892	49
12	.68455	.72897	.69717	.71691	.93906	1.06489	.97246	1.02832	48
13	.68476	.72877	.69737	.71671	.93961	1.06427	.97302	1.02772	47
14	.68497	.72857	.69758	.71650	.94016	1.06365	.97359	1.02713	46
15	.68518	.72837	.69779	.71630	.94071	1.06303	.97416	1.02653	45
16	.68539	.72817	.69800	.71610	.94125	1.06241	.97472	1.02593	44
17	.68561	.72797	.69821	.71590	.94180	1.06179	.97529	1.02533	43
18	.68582	.72777	.69842	.71569	.94235	1.06117	.97586	1.02474	42
19	.68603	.72757	.69862	.71549	.94290	1.06056	.97643	1.02414	41
20	.68624	.72737	.69883	.71529	.94345	1.05994	.97700	1.02355	40
21	.68645	.72717	.69904	.71508	.94400	1.05932	.97756	1.02295	39
22	.68666	.72697	.69925	.71488	.94455	1.05870	.97813	1.02236	38
23	.68688	.72677	.69946	.71468	.94510	1.05809	.97870	1.02176	37
24	.68709	.72657	.69966	.71447	.94565	1.05747	.97927	1.02117	36
25	.68730	.72637	.69987	.71427	.94620	1.05685	.97984	1.02057	35
26	.68751	.72617	.70008	.71407	.94676	1.05624	.98041	1.01998	34
27	.68772	.72597	.70029	.71386	.94731	1.05562	.98098	1.01939	33
28	.68793	.72577	.70049	.71366	.94786	1.05501	.98155	1.01879	32
29	.68814	.72557	.70070	.71345	.94841	1.05439	.98213	1.01820	31
30	.68835	.72537	.70091	.71325	.94896	1.05378	.98270	1.01761	30

TABLE 7. TRIGONOMETRIC TABLES (CONTINUED).

31	.68857	.72517	.70112	.71305	.94952	1.05317	.98327	1.01702	29
32	.68878	.72497	.70132	.71284	.95007	1.05255	.98384	1.01642	28
33	.68899	.72477	.70153	.71264	.95062	1.05194	.98441	1.01583	27
34	.68920	.72457	.70174	.71243	.95118	1.05133	.98499	1.01524	26
35	.68941	.72437	.70195	.71223	.95173	1.05072	.98556	1.01465	25
36	.68952	.72417	.70215	.71203	.95229	1.05010	.98613	1.01406	24
37	.68983	.72397	.70236	.71182	.95284	1.04949	.98671	1.01347	23
38	.69004	.72377	.70257	.71162	.95340	1.04888	.98728	1.01288	22
39	.69025	.72357	.70277	.71141	.95395	1.04827	.98786	1.01229	21
40	.69046	.72337	.70298	.71121	.95451	1.04766	.98843	1.01170	20
41	.69067	.72317	.70319	.71100	.95506	1.04705	.98901	1.01112	19
42	.69088	.72297	.70339	.71080	.95562	1.04644	.98958	1.01053	18
43	.69109	.72277	.70360	.71059	.95618	1.04583	.99016	1.00994	17
44	.69130	.72257	.70381	.71039	.95673	1.04522	.99073	1.00935	16
45	.69151	.72236	.70401	.71019	.95729	1.04461	.99131	1.00876	15
46	.69172	.72216	.70422	.70998	.95785	1.04401	.99189	1.00818	14
47	.69193	.72196	.70443	.70978	.95841	1.04340	.99247	1.00759	13
48	.69214	.72176	.70463	.70957	.95897	1.04279	.99304	1.00701	12
49	.69235	.72156	.70484	.70937	.95952	1.04218	.99362	1.00642	11
50	.69256	.72136	.70505	.70916	.96008	1.04158	.99420	1.00583	10
51	.69277	.72116	.70525	.70896	.96064	1.04097	.99478	1.00525	9
52	.69298	.72095	.70546	.70875	.96120	1.04036	.99536	1.00467	8
53	.69319	.72075	.70567	.70855	.96176	1.03976	.99594	1.00408	7
54	.69340	.72055	.70587	.70834	.96232	1.03915	.99652	1.00350	6
55	.69361	.72035	.70608	.70813	.96288	1.03855	.99710	1.00291	5
56	.69382	.72015	.70628	.70793	.96344	1.03794	.99768	1.00233	4
57	.69403	.71995	.70649	.70772	.96400	1.03734	.99826	1.00175	3
58	.69424	.71974	.70670	.70752	.96457	1.03674	.99884	1.00116	2
59	.69445	.71954	.70690	.70731	.96513	1.03613	.99942	1.00058	1
60	.69466	.71934	.70711	.70711	.96569	1.03553	1.00000	1.00000	0
	Cosin	Sine	Cosin	Sine	Cotang	Tang	Cotang	Tang	
	46°		45°		46°		45°		

(2) To get the value of the sine, cosine, tangent, or cotangent of an angle between 0° and 45°, proceed as follows:

(a) Find the number of degrees at the top of the page of the table.

(b) Find the number of minutes in the extreme left hand column, reading from the top toward the bottom.

(c) Locate the proper column for the function (sine, cosine, tangent, or cotangent), using the headings at the top.

(d) Find the value of the function in this column at a point directly across from the given number of minutes.

EXAMPLE

Find the sine of 43° 30′; that is, sin 43° 30′ = ?

SOLUTION

Since this angle is between 0° and 45°, the degree heading as well as the function will be found at the top of the page. Use the left-hand minute column and follow down to the value of 30′. The first column of functions is used because the required function is the sine. Thus, in the "sine" column 6 at a point across from 30′, we find that sin 43° 30′ = .68835.

EXAMPLE

Find the cosine of 43° 59′; that is cos 43° 59′ = ?

SOLUTION

For angles between 0° and 45°, the value of the cosine is found in the column headed "cosine." The "minute" column is followed down to 59′, and then in the "cosine" column, at a point across from 59′, we find that cos 43° 59′ = .71954.

EXAMPLE

Find the tangent of 44° 10′; that is, tan 44° 10′ = ?

SOLUTION

For angles between 0° and 45°, the value of the tangent is found in the column headed "tangent. " The "minute" column is followed down to 10′, and then in the "tangent" column, at a point across from 10′, we find that tan 44° 10′ = .97133.

6. Interpolation

a. Interpolation is a method of estimating the value of functions of angles which are not given in the tables, or estimating the angle, given the function which is not listed in the tables. Briefly, it is a process that assumes a straight-line difference between two values, such that the sine of 43° 30′ 30″ has a value halfway between 43° 30′ and 43° 31′, and may be found by adding one-half their difference to the function of 43° 30′

EXAMPLE

Find the sine of 44° 30′ 40″.

SOLUTION

Since the sine of 44° 30′ 40″ is somewhere between the sine of 44° 30′ and sine of 44° 31′, find the value of the latter functions and subtract the value of sin 44° 30′ from the value of sin 44° 31′.

sin 44° 31' = .70112
sin 44° 30' = .70091

Difference = .00021

The desired function is 40/60 or .67 of the one minute difference.

Therefore, .67 x .00021 = .00014
sin 44° 30′ 40″ = .70091 + .000014 = .70105.

EXAMPLE

Find the cosine of 43° 20′ 20″.

SOLUTION

Find and subtract the value of the function cos 43° 21′ from the value of the function cos 43° 20′.

cos 43° 20' = .72737
cos 43° 21' = .72717

Difference = .0020

20/60 x .00020 = .00007

Actually 20/60 x .00020 = .0000666. But rounding off .0000666 to five places is .00007.

Therefore, cos 43° 20′ 20″ = .72737 - .00007 = .72730.

b. In the next example, the process varies slightly. It is necessary to subtract the difference from the value of the smaller angle. This is true in the case of all cofunctions because their values decrease as the angle increases. The process varies slightly when an angle is desired from a given function.

EXAMPLE

Find the angle whose sine is .68420.

SOLUTION

sin 43° 11′ = .68434
sin x = .68420 (unknown angle)
sin 43° 10′ = .68412

Difference between sin 43° 10′ and sin 43° 11′ = .00022.

Difference between sin 43° 10′ and sin x (unknown angle) = .00008.

From this the desired angle is $\frac{.00008}{.00022}$ of the way from 43° 10′ to 43° 11′;

$$\frac{.00008}{.00022} \times 60 \text{ (number of seconds in one minute)} = 22.$$

Therefore, the desired angle is 43° 10′ 22″.

c. To get the value of the sine, cosine, tangent, or cotangent of an angle between 45° and 90°, use the degrees at the bottom, as explained below.

(1) Find the number of degrees at the bottom of the trigonometric tables (see Tables 6 and 7, on pages 59 and 60).

(2) Find the number of minutes in the extreme right-hand column, reading from the bottom toward the top.

(3) Locate the proper column for the function, using the headings at the bottom.

(4) Find the value of the function in this column at a point directly across from the given number of minutes.

EXAMPLE

Find the sine of 46° 15′; that is, sine 46° 15′ = ?

SOLUTION

For angles between 45° and 90°, the value of the sine is found in the column marked “sine” at the bottom. The “minutes” column is followed up to 15′, and then in the “sine” column at a point across from 15′, sin 46° 15′ = .72236.

EXAMPLE

Find the tangent of 45° 48′; that is, tan 45° 48′ = ?

SOLUTION

For angles between 45° and 90°, the value of the tangent is found in the column marked “tangent” at the bottom . The “minute” column is followed up to 48′, and then in the “tangent” column at a point across from 48′, tan 45° 48′ = 1.02832.

7. Angle Corresponding to a Given Function

In the preceding examples and problems, finding the value of the trigonometric function of a given angle has been discussed. It is also necessary to understand the reverse of this procedure; that is, how to use the tables of trigonometric functions to find the angle corresponding to a given trigonometric function. The procedure is as follows:

a. Locate the given number (value of function) such as the sine of .69466 in the proper column, using tables 6 and 7 on pages 59 and 60 to find the angle of the corresponding function, as explained in the following two paragraphs.

b. When the heading is at the top of the column, the number of degrees is found at the top of the page, and the number of minutes will be in the extreme left-hand column. In this case, the angle of the sine of .69466 is 44° 0′.

c. When the beading is at the bottom of the column, the number of degrees is found at the bottom of the page, and the number of minutes will be in the extreme right-hand column. NOTE: The following summary is provided to help in locating the value of functions in the trigonometric tables. This summary shows how the functions of an angle change in value for angles from 0° to 90°. Notice that as the angle for sine increases from 0° to 90°, its value also increases from zero to 1.0000. As the angle for cosine increases from 0° to 90°, its value decreases from 1.000 to zero. As the angle for tangent increases from 0° to 90°, its value increases from zero to infinity. And, as the angle for cotangent increases from 0° to 90°, its value decreases from infinity to zero.

SUMMARY

	0°	**45°**	**90°**
Sine increases	**zero**	**.70711**	**1.0000**
Cosine decreases	**1.000**	**.70711**	**zero**
Tangent increases	**zero**	**1.0000**	**infinity**
Cotangent decreases	**infinity**	**1.0000**	**zero**

EXAMPLE

Find the value of angle A when sin A = .96923.

SOLUTION

By referring to the preceding summary, it is seen that angle A must be greater than 45° and closer to 90°. Therefore, examining the columns with the sine heading at the bottom discloses the number

0.96923 on the page marked 75° at the bottom (see Table 8). In this case, the minutes are found in the extreme right-hand column. The value of the minutes corresponding to the number 0.96923 is 45′. Therefore, angle A = 75° 45′.

EXAMPLE

Find the value of angle A when cos A = 0.86603.

SOLUTION

According to the preceding note, angle A must be less than 45°. The number 0.86603 is found at the 30° column marked "cosine" at the top (Table 9 on the following page). The value of the minutes, in the left-hand column, is found to be zero. Therefore, angle A = 30° 0′.

TABLE 8. TRIGONOMETRIC TABLE.

SINES AND COSINES, NATURAL

′	10°		11°		12°		13°		14°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

EXAMPLE

Find the value of angle A when tan A = 0.18384.

SOLUTION

Evidently angle A is less than 45°. Therefore, the number 0.18384 is found in the "tangent" column with the heading at the top of the 10° page (Table 10 on the following page). The value of the minutes in the left-hand column is found to be 25°. Therefore, angle A = 10° 25′

TABLE 9. TRIGONOMETRIC TABLE.

SINES AND COSINES, NATURAL

	30°		31°		32°		33°		34°		
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

TABLE 10. TRIGONOMETRIC TABLE.

TANGENTS AND COTANGENTS, NATURAL

′	8°		9°		10°		11°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

8. Solution of Right Triangles

If any two sides of any right-angled triangle are known, the third side can be calculated from the formula $c^2 = a^2 + b^2$, where "c" is the hypotenuse, and "a" and "b" are the other sides.

An easier method for the solution of the sides of a right triangle, and one which also includes the solution of the angles, is found in the use of trigonometric functions. The parts of a triangle consist of three sides and three angles. A right triangle may be solved if, in addition to the right angle, two parts are known (at least one of them being a side). The two known parts must be either one of the acute angles and any one of the sides, or any two sides.

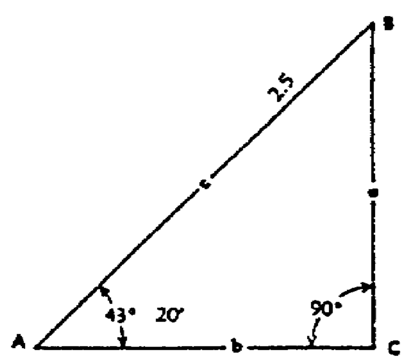
EXAMPLE

Given an acute angle and the hypotenuse in figure 18, find angle B and sides "a" and "b."

SOLUTION

Here, "a" is the side opposite and "b" is the side adjacent to Angle A. Angle B = 90°- A = 46° 40′, which is the complement of angle A. According to rule (5)(see page 53), side opposite = hypotenuse x sine. Substituting 2.5 cm for hypotenuse and .6862 for sine, side opposite = 2.5 x .6862- = 1.716 cm. According to rule (8)(see page 53), side adjacent = hypotenuse x cosine. · Substituting 2.5 cm for hypotenuse and .7274 for cosine, side adjacent = 2.5 x .7274 = 1.819.

FIGURE 18. FIND ANGLE "B" AND SIDES "A" AND "B."



EXAMPLE

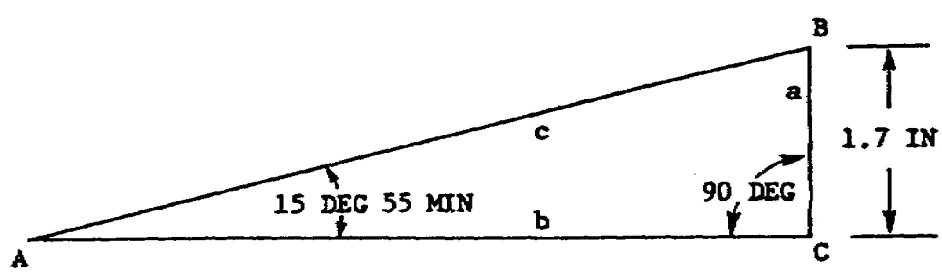
Given an acute angle and the opposite side in figure 19, find angle B and sides "b" and "c."

SOLUTION

Here, side "b" is the adjacent side to angle A and "c" is the hypotenuse. Angle B = 90° - A = 74° 5′. According to rule (9)(see page 53), side adjacent = side opposite x cotangent. Substituting 1.7 inch for side opposite and 3.5067 for cotangent, side adjacent = 1.7 x 3.5067 = 5.961 inch. According to rule 11, hypotenuse = side opposite + sine. Substituting 1.7 inch for side opposite and .2742 for sine, side opposite =

$$\frac{1.7}{.2742} = 6.1999 \text{ in}$$

FIGURE 19. FIND ANGLE B AND SIDES "B" AND "C."



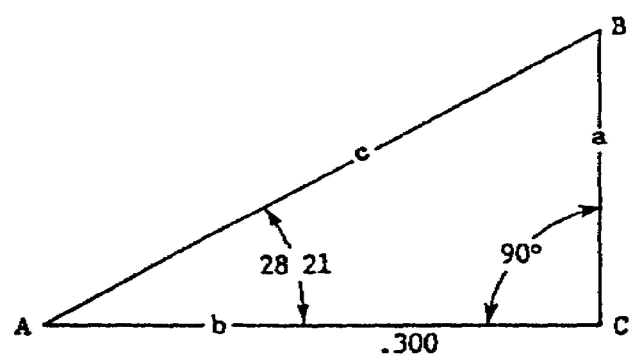
EXAMPLE

Given an acute angle and adjacent side in figure 20, find angle B and sides "a" and "c."

SOLUTION

Here, "a" is the side opposite and "c" is the hypotenuse. Angle B = 90° - A = 61° 39'. According to rule (6)(see page 53), side opposite = side adjacent ÷ tangent, or rule (7)(see page 53), side opposite = side adjacent ÷ cotangent. Substituting .300 meters for side adjacent and .5396 for tan, side opposite = .300 x .5396 = .1619 meter. Or, substituting .300 for side adjacent and 1.8533 for cotangent, side opposite = .300 ÷ 1.8533 = .1619. According to rule (12)(see page 54), hypotenuse side adjacent ÷ cosine. Substituting .300 for side adjacent and .8801 for cosine, hypotenuse = .300 ÷ .8801 = .34087.

FIGURE 20. FIND ANGLE B AND SIDES "A" AND "C."



EXAMPLE

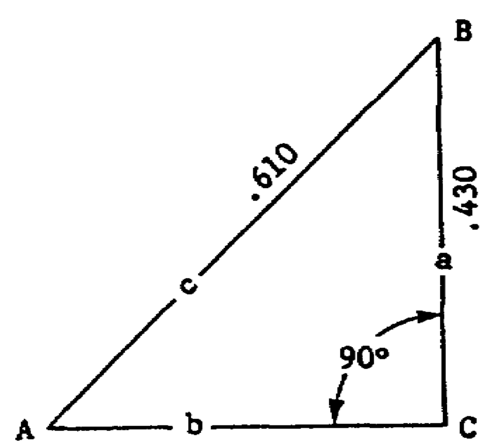
Given the hypotenuse and one side, find the angles A and B, and side "b" of figure 21.

SOLUTION

Here, "b" is the side adjacent. According to rule (1)(see page 53), sin A = side opposite ÷ hypotenuse. Substituting .430 for side opposite and .610 for hypotenuse, sin A = .430 ÷ .610 = .70492.

Therefore, A = 44° 49′ 23″ and B = 90° - A = 45° 10′ 37″. According to rule 8, side adjacent = hypotenuse x cosine. Substituting .610 for hypotenuse and .7093 for cosine, we get: side adjacent = .610 x .7093 = 4327.

FIGURE 21. FIND ANGLES A AND B, AND SIDE "B."



EXAMPLE

Given two sides (figure 22 on the following page), find angles A and B, and side "c."

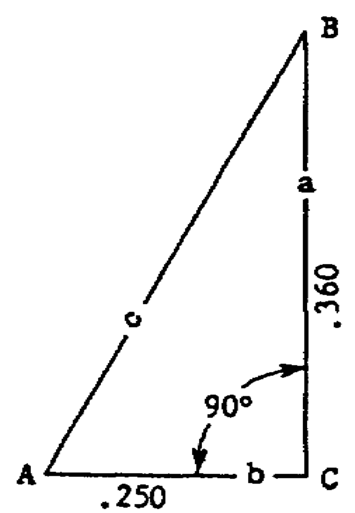
SOLUTION

According to rule (3)(see page 53), tan A = side opposite ÷ side adjacent. Substituting .360 for side opposite and .250 for side adjacent, tan A = .360 ÷ .250 = 1.44; hence, angle A = 55° 13′ 20″, angle B = 34° 46′ 40″. According to rule 12, hypotenuse = side adjacent ÷ cosine.

Substituting .250 for side adjacent and .5704 for cosine: hypotenuse = .250 ÷ .5704 = .4383.

This concludes the processes for solving right triangles. In the following paragraphs, the process for solving special right triangles will be described. Here, the sum of all the angles is 180°, as in the right triangles previously discussed. Special right triangles, however, are triangles such as the isosceles triangle, which has two 45° angles at opposite ends from each other, with the third angle equaling 90° and, therefore, has two sides that are of equal length. Thus, 45° + 45° + 90° = 180°. The other special type of triangle is that which has a 30° angle and a 600 on opposite ends from each other, with the third angle equaling 90°. Thus, 30° + 60° + 90° = 180°.

FIGURE 22. FIND ANGLES A AND B, AND SIDE "C."



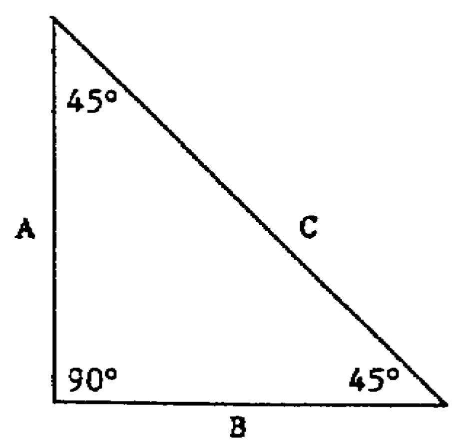
9. Special Right Triangles (45° - 45°; 30° - 60°)

a. The isosceles right triangles (two equal sides and two equal angles) and triangles with a 30° angle and a 60° angle are referred to in the machine shop as "special right triangles" because of the formulas which can be derived from the relationship of their sides and angles. These relationships enable one to make certain substitutions in the general right triangle rule formula, and to derive certain constants which hold true no matter what the size the right triangle is, just as long as its angles are 45° - 45° or 30° - 60°.

b. Derivation of the 45° - 45° Isosceles Triangle Relationship.

(1) In a 45° - 45° right triangle, as in any isosceles triangle, the sides opposite the equal angles are equal. Thus, in figure 23, side A can be substituted for side B. For example, if side A equals 2 inches, side B would also equal 2 inches; therefore, the length of the hypotenuse, side C, could be determined by multiplying the square root of one side by a value of 2. In the following example we will show how the length of side C, the hypotenuse, is derived by using side A as described above. The last two steps in this procedure serve to demonstrate that once the length of side C has been found, the length of the two opposite sides can be determined by multiplying the length of the hypotenuse or side C by the sine of either of the 45° angles. The following example demonstrates this process.

FIGURE 23. FUNCTIONS OF A 45° ANGLE.



EXAMPLE

Given: Sides A and B both equal 2 inches in length.

Therefore:

$$\sqrt{C^2} = \sqrt{A^2 + B^2}$$
$$\sqrt{C^2} = \sqrt{4 + 4}$$
$$= \sqrt{8}$$
$$C = 2.83 \text{ inches}$$

To confirm the relationship:

side A = hypotenuse x sine of 45°

= 2.83 x .707

= 2 inches

(2) Thus, in every 45° - 45° right triangle, each of the other two sides is always equal to .707 x hypotenuse.

c. Derivation of the 30° - 60° Triangle Relationship.

(1) To understand the derivation of this relationship, it must be remembered that in a right triangle the sine of 30° equals .5000 (see Table 9 on page 67) or 1/2, and also equals the side opposite the 30° angle divided by the hypotenuse. Therefore, the side opposite the 30° angle divided by the hypotenuse is equal to 1/2, or the side opposite the 30° angle is equal to 1/2 times the hypotenuse.

EXAMPLE

In figure 24 on the following page, let side A = 5 inches and side C = 10 inches. According to Rule (1)(see page 53),

$$\text{sine} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

Therefore:

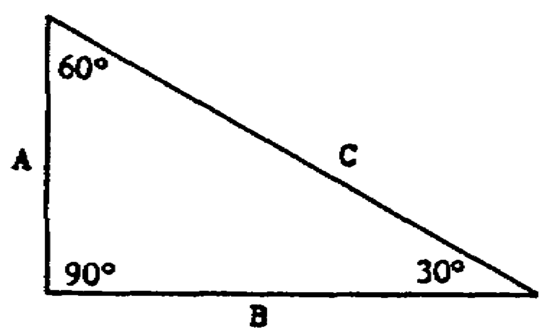
$$\sin 30^{o} = \frac{5 \text{ inches}}{10 \text{ inches}} = .5'' \text{ or } 1/2''$$

According to Rule (5)(see page 53), the side opposite = hypotenuse x sine.

Therefore:

Side A (the side opposite the 30° angle)	= 10 inches x .5 (sine of the 30° angle)
	= 5 inches (or 1/2 the length of the hypotenuse)

FIGURE 24. RELATIONSHIP BETWEEN 30° AND 60° ANGLES.



(2) In figure 24, A is the side opposite the 30° angle, C is the hypotenuse, and B is the adjacent side. Sine 60° equals .866 (Table 11 on the following page); and since sine 60° = side opposite + hypotenuse = B/C, then B/C = .866. Therefore, B = .866 x C, the hypotenuse. Thus, the following relationship is derived: side opposite 60° angle = .866 x hypotenuse.

TABLE 11. TRIGONOMETRIC TABLES.

SINES AND COSINES, NATURAL

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

EXAMPLE

In figure 24, on the previous page, let side C = 10 inches and side B = 8.66 inches. According to Rule (1)(see page 53),

$$\text{sine} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

Therefore:

$$\sin 60^\circ = \frac{8.66 \text{ inches (side B)}}{10 \text{ inches (side C)}}$$

$$= .866$$

According to Rule (5)(see page 53), the side opposite = hypotenuse x sine.

Therefore:

side B (the side opposite the 60° angle)	=	10 inches (length of side C, the hypotenuse)	x	.866 (sine of the 60° angle)
	=	8.66 inches		

10. Practical Applications

The cutting of regular polygons, such as hexagons and squares, is common practice in shop work. The constants just discussed are put to use here.

EXAMPLE

Find the distance across the flats of the largest hexagon which may be cut from a 15 inch (or 15 cm) diameter bar of round mild steel stock.

SOLUTION

A hexagon is a polygon bounded by six flat sides. Each flat side is the opposite side of a 60° angle. The 15 inch (or 15 cm) diameter of the round stock is the hypotenuse of each of six 60° angles in the round stock when viewed from either end.

In paragraph 9c(2) on page 75, and Table 11 on the previous page, we found that sine 60° = .866.

According to Rule (5)(see page 53), the opposite side = hypotenuse x sine.

Therefore:

Distance across flats (opposite side) = diameter of stock x .866

Distance across flats (opposite side) = 15 in x .866 or 15 cm x .866

= 12.99 in or 12.99 cm

EXAMPLE

Find the diameter of round bar stock required to cut a hexagon 9 cm across flats.

SOLUTION

$$\text{Diameter of stock} = \frac{\text{distance across flats}}{.866}$$

$$= \frac{9 \text{ cm}}{.866}$$

$$= 10.39 \text{ cm}$$

EXAMPLE

Find the largest square which may be cut from a 13 cm diameter bar of round stock.

SOLUTION

Largest square = diameter of stock x .707

= 13 cm x .707

= 9.191 cm

EXAMPLE

Find the diameter of round stock required to cut a square of 10 cm.

SOLUTION

$$\text{Diameter of stock} = \frac{\text{square}}{.707}$$

$$= \frac{10\text{ cm}}{.707}$$

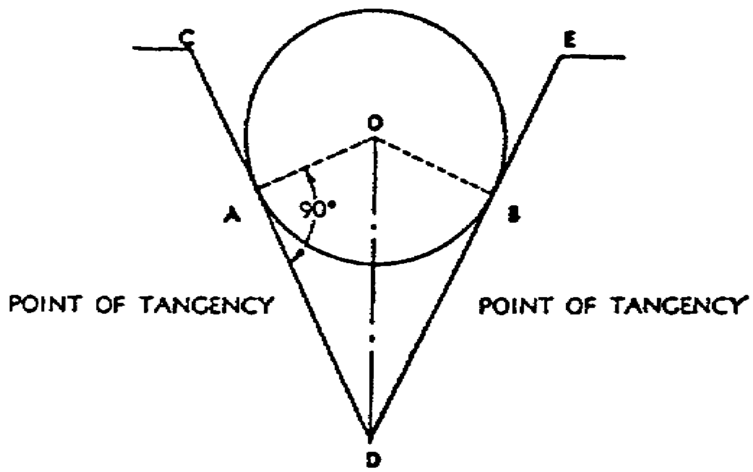
$$= 14.14\text{ cm}$$

11. General Procedure for Solving Problems in Trigonometry

a. It is relatively easy to solve a shop problem when the necessary right triangle is immediately obvious. But in actual practice, problems arise which involve shapes other than right triangles. In such cases, it is necessary to resolve the problem into right triangles by means of connecting lines. This procedure is called “triangulation”. Sometimes as many as four right triangles must be constructed in order to figure a desired value. Where any difficulty is experienced in recognizing the elements of the necessary right triangles, it is advisable to construct a diagram three or four times larger than actual size, according to scale.

b. In many problems involved in tool work, exact measurements can be obtained only by the use of accurately ground plugs. The solution of such problems involves several principles of layout work which must be thoroughly understood. A plug inserted in an opening can be represented by a circle touching two surfaces, as shown in figure 25.

FIGURE 25. LAYOUT OF PLUG INSERTED IN OPENING.

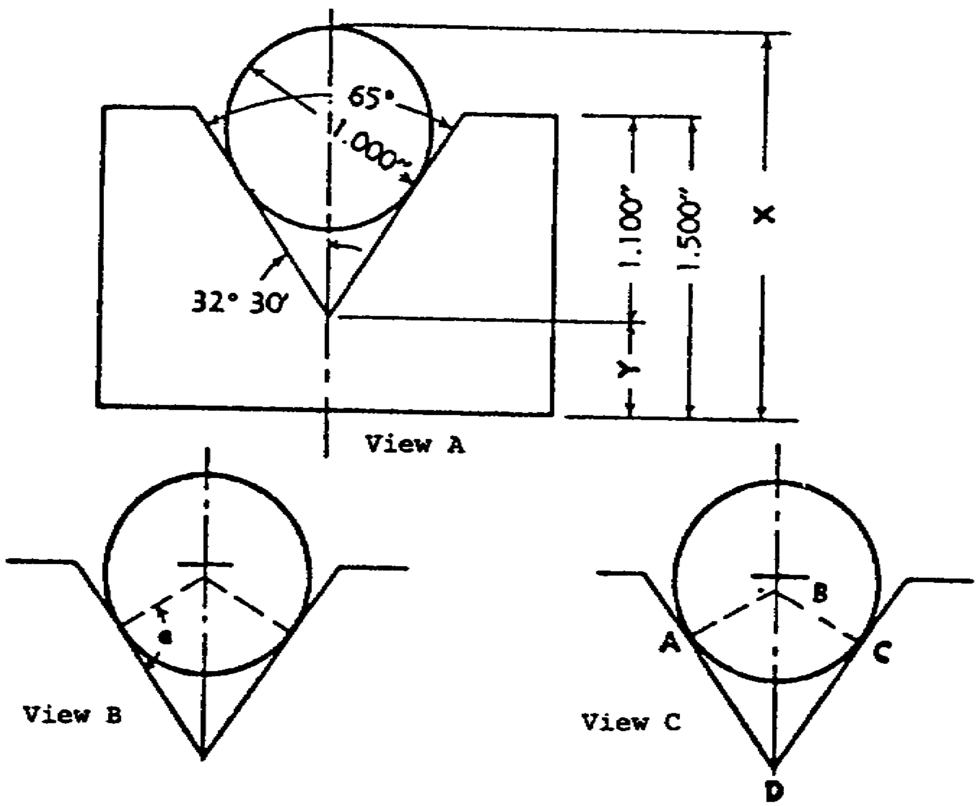


The points of contact, A and B are called the "points of tangency". A tangent to a circle is perpendicular to the radius at the point of tangency; thus, $\overline{DC}$ makes a right angle with radius $\overline{AO}$. Two tangents drawn from the same point to a circle are equal; hence, $\overline{DA} = \overline{DB}$. A line, joining the center of a circle with the intersection of two tangents, bisects the angle between the tangents; thus, line $\overline{OD}$ bisects angle ADB.

EXAMPLE

Find the dimension of X in figure 26, view A, on the following page.

FIGURE 26. FIND THE DIMENSION OF "x."



SOLUTION

Step 1. Inspect the problem to determine which sides or angles are known. According to figure 26, view A, there are two 32° 30' angles. The diameter of the circle (plug) is 1 inch.

Step 2. Generally, the first distance which must be found is that from the center of the plug to the vertex of the angle.

Step 3. To do this, it is necessary to find a right triangle which includes this distance. First, find the point of tangency on both sides of the slot by constructing a radius line at right angles to the side of the slot, as shown in figure 26, view B.

Step 4. Then construct ABCD as shown in figure 26, view C. The center line of the slot bisects angle D, and divides the figure into two equal right triangles ABD and CBD. The angle at D in each triangle has a value of 32° 30′(one-half the value of 65°), shown in figure 26, view A, on the previous page.

Step 5. Thus, the line $\overline{DB}$ (from the center of the plug to the vertex) is the hypotenuse. According to Rule (11), page 54, hypotenuse = opposite side ÷ sine. Thus:

$$\text{hypotenuse} = \frac{.5}{.53730}$$

$$= .9305788 \text{ inch}$$

Step 6. Subtract the height of the slot from the height of the block to get the dimension "Y." Thus 1.500 - 1.100 = .400.

Step 7. Add the values line DB (hypotenuse), .9305788 inch, .400 inch (the distance of "Y"), and .5 inch, the radius of the circle (plug), as follows:

Value of hypotenuse	**.9305788**
Distance of "Y"	**.400**
Radius of circle	**.5**
Thus, the value of X is:	**1.8305788**
or 1.8306	

12. Law of Sines

a. In the oblique triangle shown in figure 27, view A, on page 83, h is perpendicular to $\overline{AB}$.

(1)
$$\sin A = \frac{h}{b} \quad \text{(Rule (1), page 53)}$$

(2)
$$\sin B = \frac{h}{a} \quad \text{(Rule (1), page 53)}$$

Divide (1) by (2):

$$\frac{\sin A}{\sin B} = \frac{\frac{h}{b}}{\frac{h}{a}} = \frac{\cancel{h}}{b} \times \frac{a}{\cancel{h}} = \frac{a}{b}\ ;\ \text{therefore } \frac{\sin A}{\sin B} = \frac{a}{b}$$

The following procedure shows the similarity:

$$\frac{\sin C}{\sin B} = \frac{c}{b}$$

Reference: The sine of (A ± B) theorem:

sin (A ± B) = sin A cos B cos ± sin B

Using sin C = sin A cos B + cos A sin B

$$\sin C = \left[\frac{h}{b} \times \frac{\overline{DB}}{a}\right] + \left[\frac{\overline{AD}}{b} \times \frac{h}{a}\right]$$ Entering the values from figure 27, view A

$$\sin C = \frac{h\,(\overline{DB} + \overline{AD})}{ab}, \text{ but } (\overline{DB} + \overline{AD}) \text{ is side c.}$$

Therefore:

$$\sin C = \frac{h\ c}{a\ b}, \text{ and } \frac{h}{a} = \sin B$$ Substituting c for $(\overline{DB} + \overline{AD})$

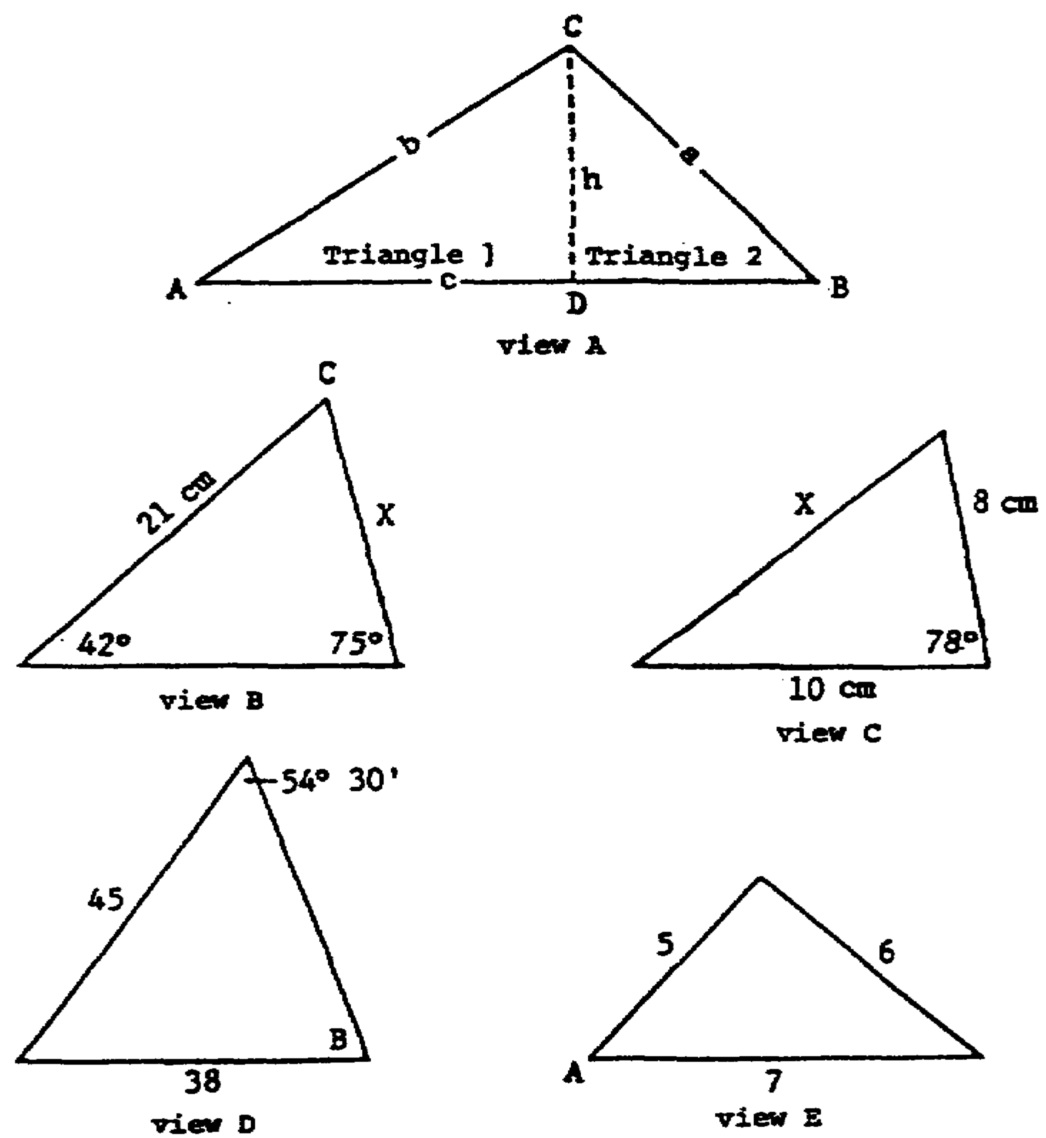
$$\sin C = \frac{\sin B \times c}{b}$$ Substituting $\frac{h}{a}$ with sin B

$$\frac{\sin C}{\sin B} = \frac{\cancel{\sin B} \times c}{b \times \cancel{\sin B}}$$ Dividing by sin B and canceling where possible

Therefore:

$$\frac{\sin C}{\sin B} = \frac{c}{b}$$

FIGURE 27. SOLVE USING THE LAWS OF SINE, COSINE, AND TANGENT.



b. The terms in the equations presented in a. above can be rearranged in the form:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Proving that $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ **(The law of sines)**

(1) Since,

$$\frac{\sin A}{\sin B} = \frac{a}{b},$$ **from paragraph 12a, page 81**

$$\frac{b \times \sin A}{\sin B} = \frac{a \times \cancel{b}}{\cancel{b}}$$ **Multiplying by b and canceling**

$$\frac{b \times \cancel{\sin A}}{\cancel{\sin A}\ \sin B} = \frac{a}{\sin A}$$ **Dividing by sin A and canceling**

Then: $$\frac{b}{\sin B} = \frac{a}{\sin A}$$

(2) And since,

$$\frac{\sin C}{\sin B} = \frac{c}{b},$$ **from paragraph 12a, page 81**

$$\frac{b \times \sin C}{\sin B} = \frac{c \times \cancel{b}}{\cancel{b}}$$ **Multiplying by b and canceling**

$$\frac{b \times \cancel{\sin C}}{\cancel{\sin C} \times \sin B} = \frac{c}{\sin C}$$ **Dividing by sin C and canceling**

Then,

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

(3) Therefore: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The above is known as the "law of sines," and should be interpreted: "Any side divided by the sine of the angle opposite is equal to any other side divided by the sine of the angle opposite it." This law, and the laws and formulas in the following paragraphs, are useful in solving oblique triangles.

EXAMPLE

Solve the triangle of figure 27, view B (on page 83) for angle C and side X using the Law of sines.

SOLUTION

Angle C + 42° + 75° = 180°, angle C + 117 = 180°; therefore, angle C = 63°.

$$\frac{X}{\sin 42^{\circ}} = \frac{21}{\sin 75^{\circ}};$$

$$X = \frac{21 \sin 42^{\circ}}{\sin 75^{\circ}}$$

$$= \frac{21 \times .66913}{.96593} = \frac{14.05173}{.96593}; \quad \text{therefore,}$$

$$X = 14.547 \text{ cm}$$

EXAMPLE

Solve the triangle in figure 27, view C (on page 83), for angle B using the Law of sines.

$$\frac{45}{\sin B} = \frac{38}{\sin 54^\circ\ 30'}, \quad \sin B = \frac{45 \sin 54^\circ\ 30'}{38},$$

$$\sin B = \frac{45 \times .81412}{38} = \frac{36.6354}{38} = .96409$$

To find angle B interpolate as follows:

$$\begin{aligned} \sin 74^\circ\ 36' &= .96410 \\ \sin 74^\circ\ 35' &= \underline{.96402} \\ \text{Difference} &= .00008 \end{aligned}$$

$$\begin{aligned} \sin x &= .95409 \\ \sin 74^\circ\ 35' &= \underline{.96402} \\ \text{Difference} &= .00007 \end{aligned}$$

From this the desired angle is $\dfrac{.00007}{}$ of the way from 74° 35′ to 74° 36′;

$$\frac{.00007}{.00008} \times 60 \ \left(\begin{array}{l}\text{number of} \\ \text{seconds in} \\ \text{1 minute}\end{array}\right) = 53 \text{ seconds}$$

Therefore, angle B is:

74° 36′ 53″

13. Law of Cosines

a. According to the Pythagorean Theorem, $c^2 = a^2 + b^2$

(1) From figure 27, view A, triangle 1, $\overline{AD} = c - \overline{DB}$. $\overline{AD}$ must be squared for use in this equation.

$b^2 = \overline{AD}^2 + h^2$

(2) Squaring of AD procedure follows:

$$\overline{AD}^2 = (c - \overline{DB})^2$$
$$\overline{AD}^2 = (c - \overline{DB})(c - \overline{DB})$$
$$\overline{AD}^2 = c^2 - c\overline{DB} - c\overline{DB} + \overline{DB}^2$$
$$\overline{AD}^2 = c^2 - 2c\overline{DB} + \overline{DB}^2$$

(3) Substituting $\overline{AD}^2$ into equation at (1) above:

$$b^2 = c^2 \quad 2c\overline{DB} + \overline{DB}^2 + h^2$$

(4) Equation for triangle 2, figure 27, view A (on the previous page):

$$a^2 = h^2 + \overline{DB}^2$$

(5) Finding the value of $\overline{DB}^2$:

$$\overline{DB}^2 + h^2 = a^2$$
$$\overline{DB}^2 + h^2 - h^2 = a^2 - h^2$$
$$DB^2 = a^2 - h^2$$

(6) Substituting DB^2 into equation at (3) above:

$$b^2 = c^2 - 2c\overline{DB} + a^2 - \cancel{h^2} + \cancel{h^2}$$
$$b^2 = c^2 - 2c\overline{DB} + a^2$$

According to the law of cosines,

$$b^2 = a^2 + c^2 - 2ac \text{ Cos } B.$$

b. Since the result of the equation at (6) above and the law of cosines both contain b^2 , both equations, therefore, are equal.

Therefore:

(1) $c^2 - 2c\overline{DB} + a^2 = a^2 + c^2 - 2ac \cos B$

(2) Subtracting by $-a^2 - c^2$:

$$-\cancel{a^2} - \cancel{c^2} + \cancel{c^2} - 2cDB + \cancel{a^2} =$$
$$\cancel{a^2} + \cancel{c^2} - 2ac \cos B - \cancel{a^2} - \cancel{c^2}$$

(3) $-2c\overline{DB} = -2ac \cos B$

(4) Dividing by -2ac:

$$\frac{-\cancel{2}\cancel{c}\overline{DB}}{-\cancel{2}a\cancel{c}} = \frac{-\cancel{2ac}\cos B}{-\cancel{2ac}}$$

(5) $$\frac{\overline{DB}}{a} = \cos B$$

(6) Multiplying both sides by a:

$$a \times \frac{DB}{a} = \cos B \times a = DB = a \cos B$$

The law of cosines is interpreted as: "The square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of those two sides and cosine of the included angle."

EXAMPLE

Solve the triangle of figure 27, view D (on page 83) for X using the Law of cosines.

SOLUTION

From the law of cosines:

$c^2 = a^2 + b^2 - 2ab \cos C$

Given:
$x = c$
$8 = a$
$10 = b$
$\cos C = \cos 78°$

$X^2 = 8^2 + 10^2 - (2 \times 8 \times 10 \cos 78°)$

$X^2 = 64 + 100 - (2 \times 8 \times 10 \times 0.20791)$

$x^2 = 64 + 100 - 33.266$

$X = 11.434$ cm

EXAMPLE

Solve the triangle of figure 27, view 3 (on page 83), for angle A using the Law of cosines.

SOLUTION

$a^2 = b^2 + c^2 - 2bc \cos A$

Given: $a = 6$
$b = 5$
$c = 7$

$6^2 = 5^2 + 7^2 - (2 \times 5 \times 7 \cos A)$

$36 = 25 + 49 - 70 \cos A$

$36 = 74 - 70 \cos A$

Subtracting by -74:

$$-74 + 36 = \cancel{-74} + \cancel{74} - 70 \cos A$$

$$-38 = -70 \cos A$$

$$-\cos A = -38$$

$$\frac{\cancel{-70} \cos A}{\cancel{-70}} = \frac{\cancel{-}38}{\cancel{-}70}$$

Dividing 38 by 70:

$\cos A = 0.54286$

By interpolation:

angle $A = 57° 7' 18''$

14. Law of Tangents

When the sides of a triangle are expressed in several figures, it is more efficient to use the following formula:

$$\frac{a - b}{a + b} = \frac{\tan 1/2\ (A - B)}{\tan 1/2\ (A + B)}$$

which is known as the "law of tangents"; a and b are any two sides. A and B are the angles opposite those sides.

EXAMPLE

Find angles A and B in figure 28, view A (on the following page) using the Law of tangents.

$$A + B + 57^\circ\ 20' = 180^\circ$$

$$A + B = 180^\circ - 57^\circ\ 20' = 122^\circ\ 40'$$

$$1/2(A + B) = 61^\circ\ 20'$$

$$\frac{9.73 - 6.47}{9.73 + 6.47} = \frac{\tan 1/2\ (A - B)}{\tan 61^\circ\ 20'}, \text{ (law of tangents)}$$

$$\tan 1/2\ (A - B) = \frac{3.26 \tan 61^\circ\ 20'}{16.20}$$

$$= \frac{3.26 \times 1.82906}{16.20}$$

$$1/2\ (A - B) = 20^\circ\ 12'\ 26'' \text{ but}$$

$$1/2\ (A + B) = 61^\circ\ 20'$$

$$A = 81^\circ\ 32'\ 26'' \text{ (adding)}$$

$$B = 41^\circ\ 7'\ 34'' \text{ (subtracting)}$$

15. Area of Triangles

a. The area of a triangle is expressed by the formula A (area) = 1/2bh, when b is the base and h is the altitude. When two sides and included angle are given (figure 28, view B), the area can be obtained from the following formula, where h and a are the sides and C is the angle:

$$\frac{h}{a} = \sin C$$

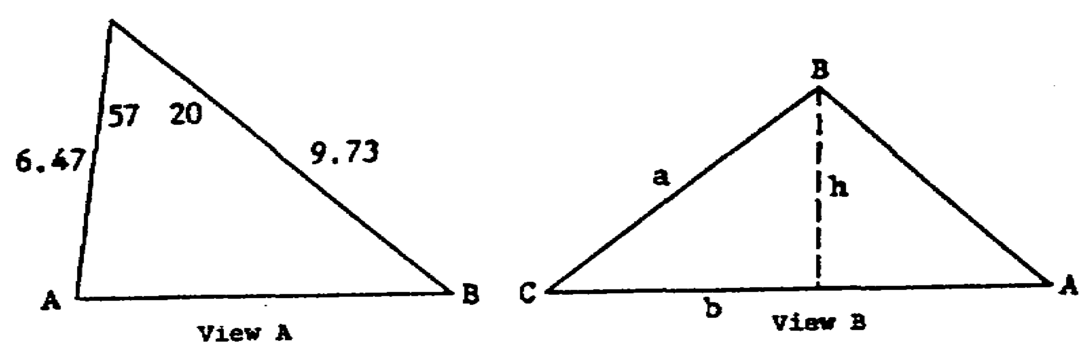
$$h = a \sin C$$

Thus, the area of the triangle may be written:

$$A = \frac{1}{2} ab \sin C$$

a and b = the two sides
Sin C = angle

FIGURE 28. FIND ANGLES AND AREA OF TRIANGLES.



b. When two angles and the included side are given (figure 28, view B, on the following page), the area can be obtained from the following formula:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a = \frac{b \sin A}{\sin B}$$

Substituting in the formula given in 15a, above, the following is obtained:

$$A = \frac{1}{2} ab \sin C \qquad \text{But } a = \frac{b \sin A}{\sin B}$$

Inserting $\frac{b \sin A}{\sin B}$, and multiplying:

$$A = \frac{1\ b \sin A \times b \sin C}{2 \sin B}$$

$$A = \frac{b^2 \sin A \sin C}{2 \sin B}$$

c. When three sides are given, the following formula is used where a, b, and c are the three sides and A = 1/2 (a + b + c), then

$$A = \sqrt{A(A - a)\ (A - b)\ (A - c)}$$

16. Conclusion

This lesson served to describe the processes for converting linear measurements from the English to metric system and from the metric to the English system, and for solving problems using ratio, proportion, and trigonometry. These mathematical processes are commonly used in all facets of machine shop operations for the fabrication of those parts, with either English or metric measurements, not normally found in the supply system. This lesson, therefore, provides a solid background in machine shop mathematics, which can also be used as a future reference to assist in solving problems encountered in daily machine shop work. At the end of this lesson, there is a practical exercise that contains problems which require computation through the use of all of the processes covered in this lesson.

PRACTICAL EXERCISE 2

1. Instructions

Read the scenario and respond to the requirements that follow the scenario.

2. Scenario

In practical exercise 1 you developed a list of mathematical problems involving the addition, subtraction, multiplication, and division of fractions and decimals; and conversion of fractions to decimals and decimals to fractions. You have administered these problems to the two former machinists who will be used as a second shift in the machine shop. Now you want to test their knowledge of machine shop calculations involving the conversion of linear measurements from the English to the metric system and vice-versa, and the solving of problems using ratio, proportion, and trigonometry. You have, therefore, developed a list of mathematical problems that you feel will assist you in determining their knowledge.

3. Requirement

Below is the list of mathematical problems that you have developed. Prepare an answer sheet by solving these problems.

a. If 2.54 centimeters equals 1 inch, how many centimeters are there in a piece of flat metal stock that is 3 yards 1 foot and 6 inches long?

b. at is the length, in inches, of a piece of round stock that is 3 meters and 15 centimeters long?

c. brass rod was cut into five lengths: 4 1/4 inch, 3 1/2 inch, 6 1/2 inch, 18 3/4 inch, and 6 3/4 inch. How long was the rod in centimeters if 1/8 inch was wasted in each cut?

d. Use the rules of conversion in paragraphs 3b and 3c, Lesson 2, Task 1 and solve the following problems:

(1) Convert 1/32 inch to millimeters.

(2) Convert 304.801 millimeters to inches.

e. If two gears have 180 and 40 teeth respectively, what is the ratio of the numbers of teeth?

f. Divide 80 trucks between two sergeants in the ratio of 5 to 3.

g. The efficiency of a machine is commonly stated as being the ratio of the output to the input (E = O/I). E = efficiency, O = output, and I = output. Suppose the input in a motor is 7000 watts and the output is 6500 watts. What is the efficiency of this motor?

h. Use the rules of proportion to solve the following problems:

(1) 70 : 45 :: 45 : X

(2) 3 1/4 : 7 4/5 :: 15 : X

(3) X : 5.6 :: 45 : 125

i. Study the gear train in figure 29 on the following page and find the rpm of the 36-tooth gear.

j. A certain single-thread worm makes 25 revolutions-per-second. It turns a worm wheel that has 27 teeth. How many revolutions-per-minute will this worm gear make?

k. Solve for the following sides of the right triangle:

(1) Find side a if sin A = 0.8, and side c = 18.5 cm.

(2) Find side b if cos A = 0.35 and side c = 4.25 cm.

(3) Find side a if tan A = 1.902 and side b = 3.75 cm.

1. Use the instructions for calculation of angles in paragraph 4, Lesson 2, Task 3 (on page 56) to solve the following problems:

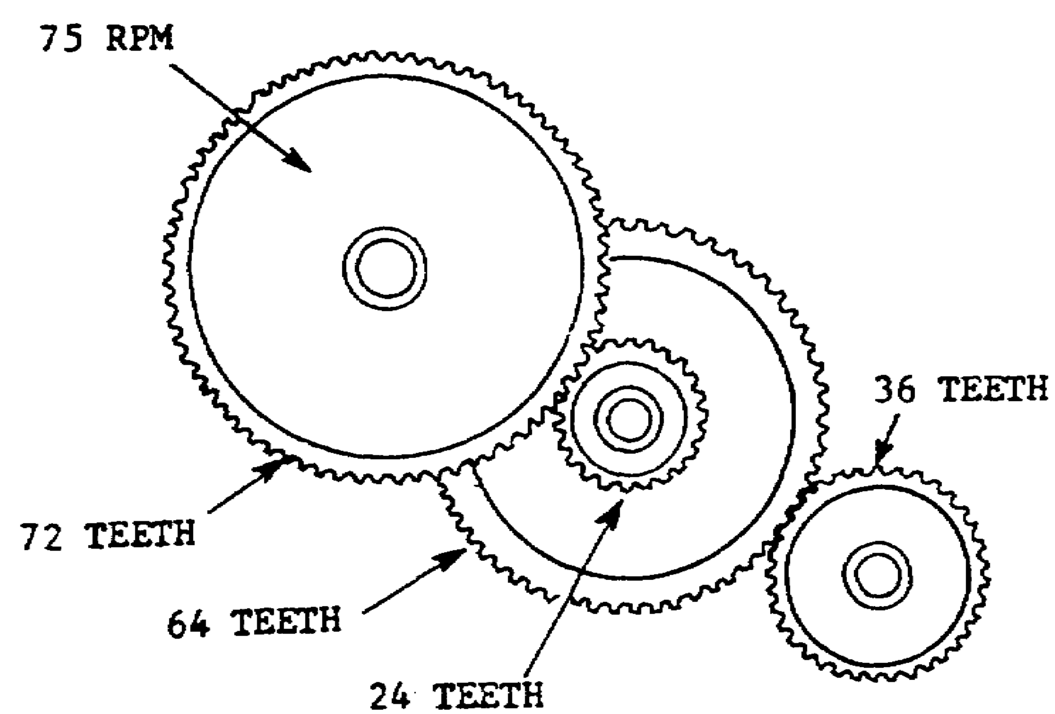
(1) Add:

(2) Subtract: 48° 32' 12"
30° 45' 20"
----

(3) Multiply: 15° 29′ 40″ by 2

(4) Divide: 85° 15′ 40″ by 2

FIGURE 29. GEAR TRAIN.



m. Use table 12 on page 98 and find the sine, cosine, and tangent of the following angles:

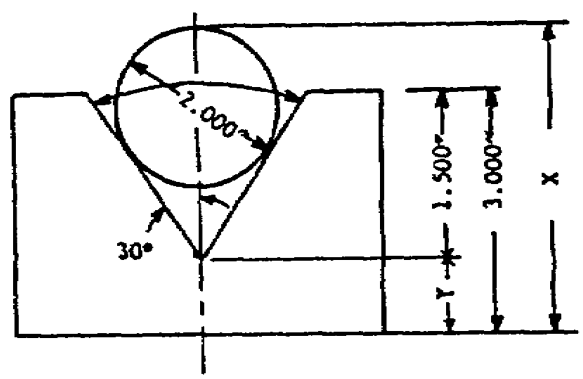
(1) 0°

(2) 1°

(3) 89° 30′

n. Find the distance X in figure 30. Use the proper trigonometric function and the trigonometric tables at table 12 to assist you in solving this problem.

FIGURE 30. FIND THE DISTANCE "X."



o. Find the sine of 44° 59′ 45″. Use the trigonometric tables at tables 6 (page 59) and 7 (page 60) to assist you in solving this problem.

p. Find the value of the following angles to the nearest minute. Use trigonometric tables 6, 7, and 12 (on the following page) to solve these problems.

(1) Sin A = .70711

(2) Cos A = .82887

(3) Tan A = .05241

(4) Cot A = 38.1885

TABLE 12. TRIGONOMETRIC TABLES.

SINES AND COSINES, NATURAL

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
29	.00844	.99996	.02589	.99966	.04333	.99906	.96076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

SINES AND COSINES, NATURAL

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

TANGENTS AND COTANGENTS, NATURAL

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

LESSON 2. PRACTICAL EXERCISE - ANSWERS

1. Requirement

a. 320.04 centimeters

b. 124 inches

c. 102.24 centimeters

d. (1) 0.79375 millimeters
(2) 12 inches

e. 4.5 to 1

f. 50 and 30

g. 93%

h. (1) 28 13/14
(2) 36
(3) 2 2/125

i. 400 rpm

j. 55.5 rpm

k. (1) 14.8 centimeters
(2) 1.4875 centimeters
(3) 7.1325 centimeters

l. (1) 116° 6′ 30″
(2) 17° 46′ 52″
(3) 30° 59′ 20″
(4) 42° 37′ 50″

m. (1) Sin 0° = 0
Cos 0° = 1.0000
Tan 0° = 0

(2) Sin 1° = .01745
Cos 1° = .99985
Tan 1° = .01746

(3) Sin 89° 30′ = .99996
Cos 89° 30′ = .00873
Tan 89° 30′ = 114.589

n. 4.5 inches

o. .70706

p. (1) 45°
(2) 34° 1’
(3) 3°
(4) 1° 30’

REFERENCES

REFERENCES

The following document was used as resource material in developing this subcourse:

FM 43-3