Coaxial Cable Delay

By: Jacques Audet VE2AZX
ve2azx@amsat.org

Introduction

Last month, I reported the results of measurements on a number of coaxial cables with the VNA (Vector Network Analyzer). (Ref. 3) This month, I describe the measurement technique and theory behind these measurements. I will also try to give an insight on what is happening.

Transmission lines are described by their two most important characteristics: the characteristic impedance Zo and the delay. For instance, take a "short" (say 0.01 wavelength) piece of coaxial cable such RG-58U and measure its capacitance with the other end open. A one foot length yields ~ 31.2 pF. The inductance may also be measured with the other end shorted. It yields ~ 76.8 nH.
The impedance may now be computed as:

$$Zo = \sqrt{\frac{L}{C}} \quad \text{Eq.(1)} \qquad Zo = \sqrt{\frac{76.8*10^{-9}}{31.2*10^{-12}}} = 49.6\ ohms$$ … Close enough to 50 ohms.

Here L and C are measured for the same length. The delay may also be computed:

$$Delay = \sqrt{L*C} \quad \text{Eq. (2)} \qquad Delay = \sqrt{76.8*10^{-9}*31.2*10^{-12}} = 1.55\,nSec$$

For an ideal line, the delay increases linearly with its length, while its impedance remains constant.

Let's compute the velocity in foot per second:

$$v = \frac{len}{delay} \quad \text{Eq. (3)} \qquad v = \frac{1}{1.55*10^{-9}} = 6.46*10^{8}$$ foot per second or $1.966*10^{8}$ meters/second

This is less than the speed of light. The ratio of the above speed to the speed of light gives us the velocity factor V_f:

$$V_f = \frac{1.966*10^{8}}{2.998*10^{8}} = 0.666$$ or 66.6 % of the speed of light.

Keep in mind that these results have been obtained with only two measurements.

As mentioned earlier, the delay increases linearly with the line length. For a given length, the phase difference between the input and output will increase with the frequency:

$$\varphi = 2\pi f * delay \quad \text{Eq. (4)}$$ Here the phase φ is in radians and the frequency f is in Hertz.

Converting the phase from radians to degrees requires multiplying by: $\frac{360}{2\pi}$

At 162 MHz, the phase delay in degrees will be:

$$\varphi \deg = f \cdot 360 * delay = 162 * 10^{6} * 360 * 1.55 * 10^{-9} \approx 90.4$$

This length that gives 90 degrees of phase shift is also known as a quarter wavelength.

Measuring Delay with the Vector Network Analyzer

The instrument measures the phase over a frequency range of interest.
In the above example, we get ~ 90 degrees of phase shift form 0 Hz to 162 MHz.
Therefore the delay may be computed from the previous equation.

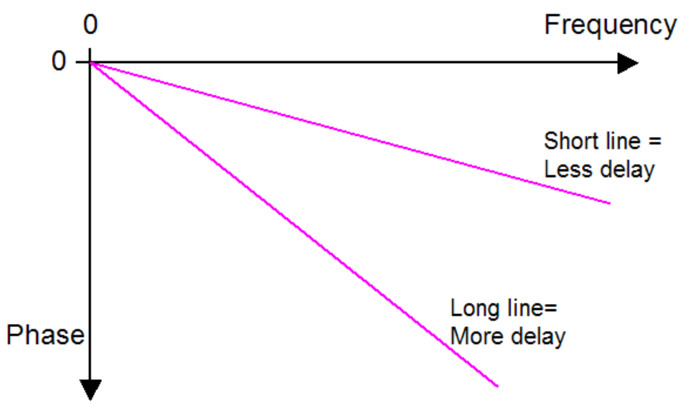

Figure 1
An ideal transmission line gives a linear change of phase versus frequency.

The slope of these phase curves gives us the delay:

$$delay = \frac{\Delta\varphi}{\Delta\omega} = \frac{\Delta\varphi * \deg}{360 * \Delta f} \qquad \text{Eq. (5)}$$

Where Δφ is the change in phase in radians, Δω the change in the frequency in radians/sec.

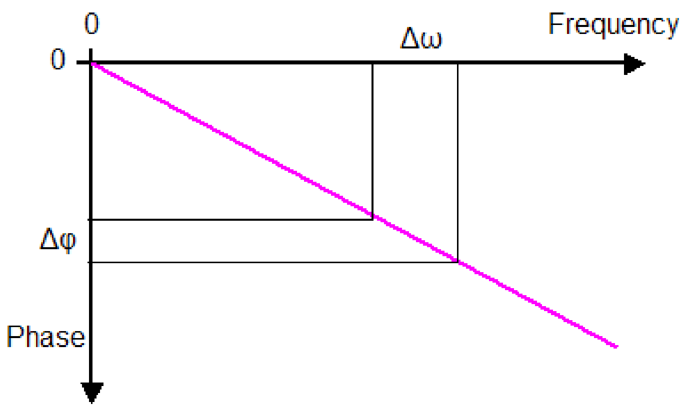

Figure 2.
The delay may be computed from the change in phase over a small frequency increment.

In practice, the phase behavior of real transmission lines is not perfectly linear. By measuring the delta phase (Δφ) for a small frequency increment (Δω), we get the actual delay over a small group of

frequencies. This actual delay is called group delay, as it gives the delay for a group of frequencies, also called the aperture. We try to use a frequency aperture as small as possible to be able to view all peaks and valleys in the group delay. This tends to give more noise in the phase measurement as the aperture is reduced. So a compromise aperture must be used that will only give a tolerable amount of noise on the group delay trace. The VNA measures the phase for a number of frequency points, such as 201 points over a span of frequencies. The frequency aperture is specified as a percentage of the frequency span. Typical aperture values range from 1 % to 10 % of span.
Figure 3 shows the phase versus frequency for a real transmission line.

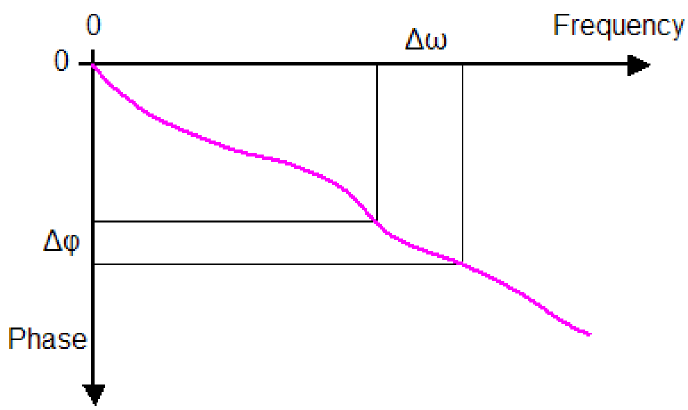

Figure 3

Delay Variations Caused by Changes in Inductance

What is causing delay variations ? Equation 2 tells us that the delay is related to the distributed inductance and capacitance of the line. These appear to be "constants" related to the line geometry and to the dielectric constant of the insulation. Remember also that the inverse of the delay (per unit length) is the velocity of propagation. The effective distributed inductance L has two components which may be considered in series: the external inductance L_e and the internal inductance L_i. The first component L_e is the inductance created by the flux outside the conductors, between the inner and the outer conductors. It is the "normal" inductance that we assume to be constant. The internal inductance L_i comes from the flux internal to the conductor. It can only exist if the conductor is not perfect. Even copper or silver is not a perfect conductor and a small part of the flux exists mainly inside the center conductor. This internal flux exists mostly where the current penetrates inside the conductor. That is at the skin depth, which decreases as the frequency increases. For instance copper has a skin depth of 2.6 mil at 1 MHz, decreasing by a factor of 1/square root(10) at 10 MHz.
At 100 MHz, the skin depth is 0.26 mil. The shin depth increases for less conductive materials. It would be zero for a perfect conductor. The effect of the skin depth is to increase the AC resistance at higher frequencies, since the current has less cross section to flow.
Here we need to know at what frequency the skin depth becomes important. That is the frequency where the DC resistance is equal to the AC resistance per unit length. For a round, non magnetic conductor, the crossover frequency f_δ in Hz, is given by:

$$f_\delta = \frac{6.2 * 10^{10}}{\pi^2 * \sigma * d^2} \qquad \text{Eq. (6)}$$

Where σ is the conductivity in siemens/m ($5.8*10^7$ for copper) and d is the diameter in inches. This equation (adapted from eq. 2.45 in ref. 1) is plotted in figure 4.

Note that below f_δ the resistance (essentially DC resistance) and the internal inductance are both constant. Computing the crossover frequency f_δ for the RG-58 cable which has a center conductor of 0.032 in. diameter gives f_δ = 106 KHz.

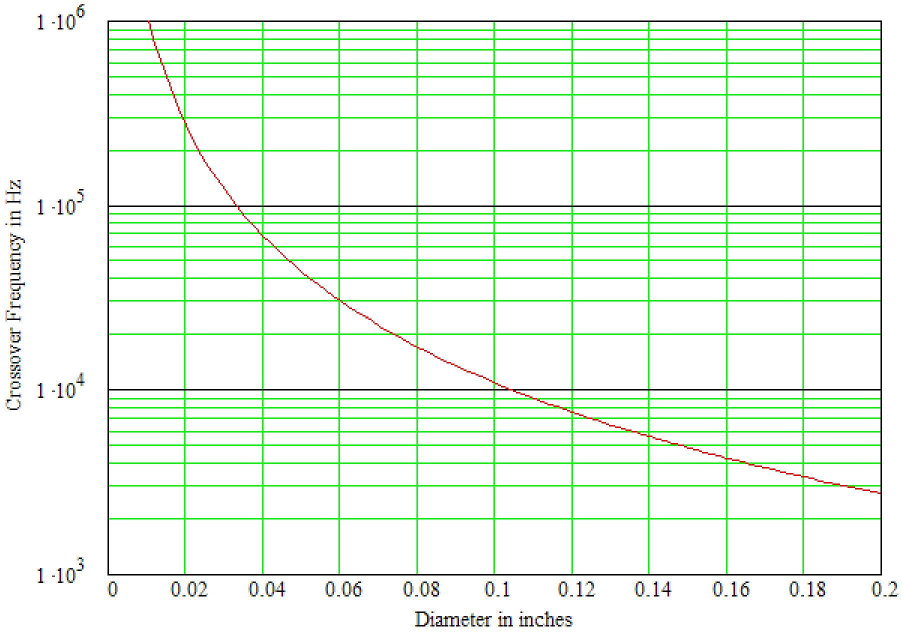

Figure 4.
Graph of equation 6, showing how the skin depth crossover frequency f_δ changes versus conductor diameter.

Refer to figure 5. Below the crossover frequency f_δ the resistance (red curve) is approx. equal to the DC resistance and the internal inductance (blue curve) is also constant at 0.05 µH. Above f_δ the AC resistance increases due to the skin effect while the internal inductance (blue curve) decreases above f_δ

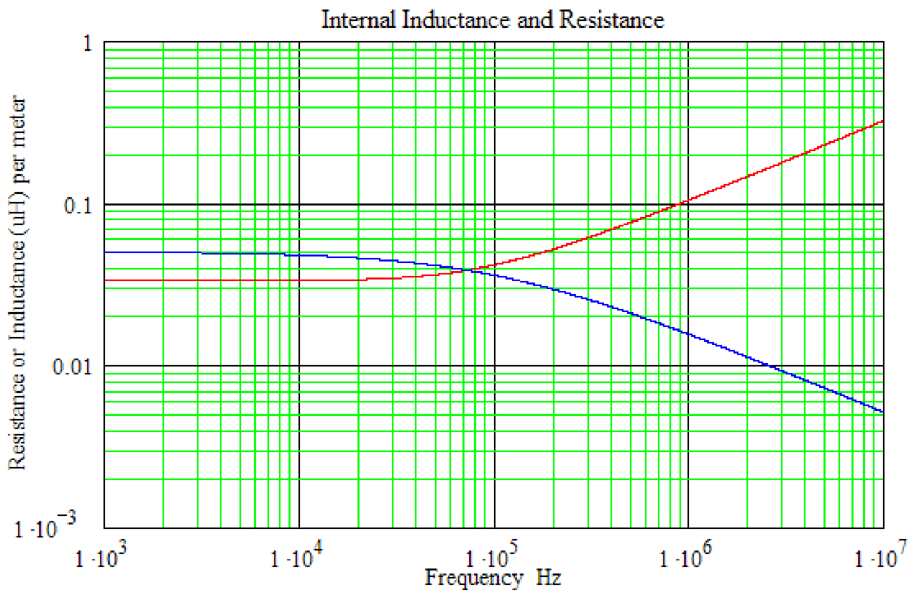

Figure 5.
Resistance and internal inductance values per meter for RG-58 cable.
Note that the internal inductance is always the same for non magnetic conductors.

Refer to the beginning of page 1, where we measured a series inductance of 76.8 nH per foot. The computed internal inductance is 50 nH / meter or 15.2 nH / foot. This is 20% of the series (external) inductance. This assumes that the measurement frequency was well above the f_δ frequency. At frequencies well below f_δ the two inductances add up, thus increasing the series inductance by 20%. From equation 1 and 2, this increase will cause an increase in the delay and line impedance by 10%. Figure 6 shows the results of my delay measurements on a length of 81 feet of RG-58 type coax cable. This coax does not have a solid core but it has a multibraid center conductor, which could modify its internal inductance.

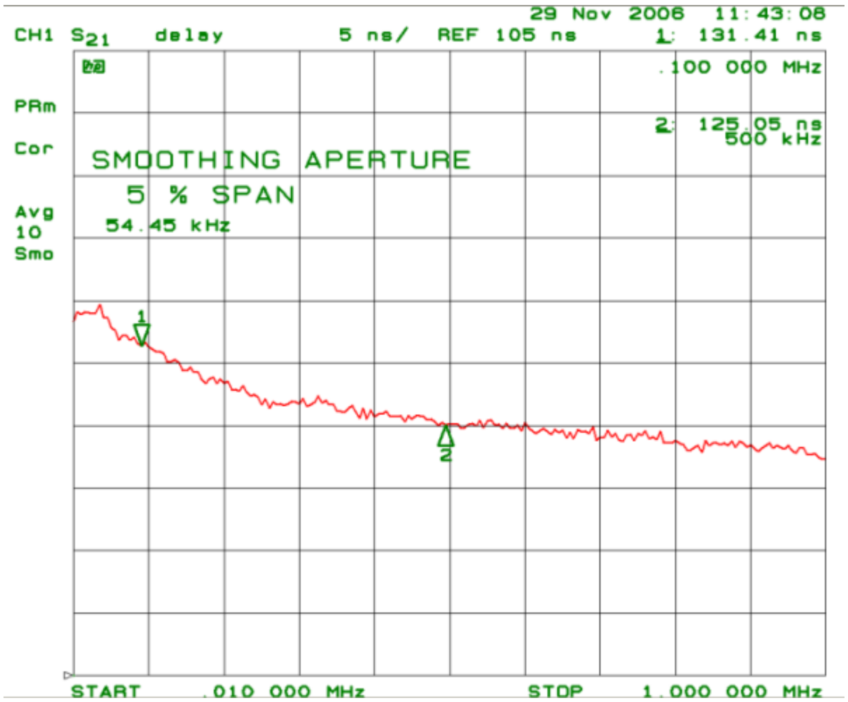

Figure 6.
Delay measurements on a 81 foot length of RG-58 coax, from 10 KHz to 1 MHz. Note that a longer line would make the measurement easier.

The expected delay variation is 10% or about 12.5 nS. Looking at the markers, the difference is 6.3 nS from 100 KHz to 500 KHz. From 10 KHz to 1 MHz, the delay change is about 12 nSec. Below approx. 50 KHz, the delay curve should not be trusted, since the VNA is only specified down to 30 KHz and the measuring aperture is 54 KHz.
Note that the delay changes more rapidly for the first 250 KHz. This could be explained by the change in internal inductance. For frequencies above 250 KHz, the delay decreases at a slower rate, possibly caused by the diminishing internal inductance and possible changes in the dielectric constant.

Line Delay above the Skin Effect Cutoff Frequency

Reference 1 (table 3.6) gives an equation that approximates the delay of long cables operated in the skin effect region:

$$delay(\omega) = \sqrt{L * C} + \frac{Ro}{4 * Zo}\sqrt{\frac{1}{\omega * \omega o}} \qquad \text{Eq. (7)}$$

Where L and C are the transmission line constants in H/m and F/m respectively, Ro is the line series AC resistance (Ω) at radian frequency ω_o, where ω_o is far above the crossover frequency f_δ. Figure 7 shows the resulting curve. There is a 5% delay variation from 1 MHz to 1 GHz. This exceeds my measured delay variations which were from 1 to 2 % over the same frequency range. The overall shape is similar, however, with the delay stabilizing above a few hundred MHz.

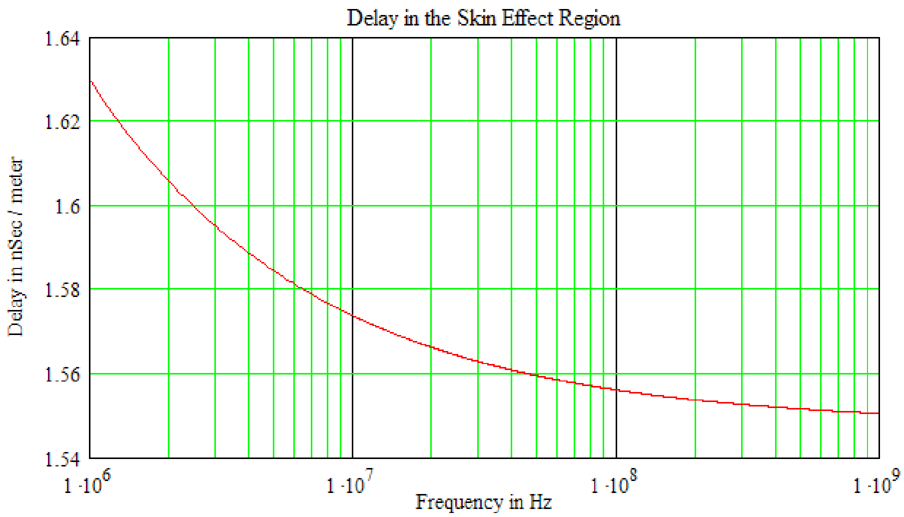

Figure 7.
Calculated delay in nSec / m for RG-58 cable.

Conclusion

The distributed inductance and capacitance are the basic transmission line parameters. From these, we can calculate the line impedance, the delay in terms of time and phase, the speed of propagation and the velocity factor. The inductive component has an additional component at the lower frequencies which slows the signal somewhat. This occurs around 100 KHz for small coax and lower for larger cables. For frequencies above 1 MHz, the dielectric constant of the cable is probably responsible for the decrease in the delay. Measuring the delay of cables can reveal some "hidden" properties that could make it unsuitable for some applications, such as carrying wideband data.

References

1- High Speed Signal Propagation – Advanced Black Magic by Dr Howard Johnson and Dr Martin Graham. A "must" for all those interested in the subject of signal integrity.

2- See Dr Howard Johnson's site: http://www.signalintegrity.com/

3- Coaxial Cable Delay Measurements from the author: http://www.geocities.com/ve2_azx/