Remote Laser Interferometry Microphone

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Abstract

This document details the theory and construction of a device that uses laser interferometry to measure the vibration of a reflective surface (which could be located outside the device itself). Although we have demonstrated that such a device is buildable and can work in the lab, there are still several issues to be dealt with before it could become usable in the field for actual measurement.

1 Operational Theory

1.1 The Stationary Michelson Interferometer

The laser interferometry microphone is based on the Michelson interferometer, shown below:

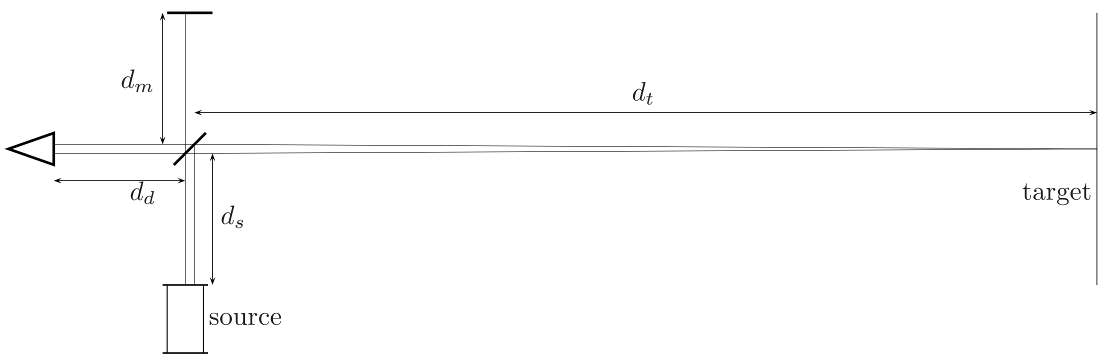

Figure 1: Schematic of the Michelson interferometer. The laser emitter, beam splitter, reflecting mirror, target, and detector are all shown, along with the approximate path of the laser beam.

The interferometer compares the relative phase of two paths of the laser light: (1) the path proceeding directly through the beam splitter, reflecting off the top mirror, and reflecting off the beam splitter to the detector, and (2) the path initially reflecting off the beam splitter, then reflecting off the target, and passing through the beam splitter to hit the detector. The two paths cover different distances:

$$d_1 = d_s + 2d_m + d_d \qquad \text{for path 1} \tag{1}$$

$$d_2 = d_s + 2d_t + d_d \qquad \text{for path 2} \tag{2}$$

Therefore, the electric fields of the two beams will probably be out of phase upon arriving at the detector. Using the sinusoidal electromagnetic wave equation:

$$\vec{E} = \vec{E}_0 \sin(kx - \omega t + \phi) \tag{3}$$

we find that for these two paths, the electric field strengths at the detector are

$$\vec{E}_1 = \vec{E}_0 \sin(kd_1 - \omega t + \phi) \tag{4}$$

$$\vec{E}_2 = \vec{E}_0 \sin(kd_2 - \omega t + \phi) \tag{5}$$

And the overall total electric field is

$$\vec{E}_d = \vec{E}_0 \big(\sin(kd_1 - \omega t + \phi) + \sin(kd_2 - \omega t + \phi)\big) \tag{6}$$

$$\vec{E}_d = \vec{E}_0 \big(\sin(kd_s + 2kd_m + kd_d - \omega t + \phi) + \sin(kd_s + 2kd_t + kd_d - \omega t + \phi)\big) \tag{7}$$

1.2 Changing Path Lengths in the Interferometer

The principle that allows the interferometry microphone to detect sound is the vibration of the target caused by ambient sound waves. This means that the path length component d_t varies in time. For later convenience, we allow d_m and $\vec{E}_d(t)$ to be time-dependent as well. We can now split the distances $d_m(t)$ and $d_t(t)$ into constant and variable components:[1]

$$d_m(t) = d_{m0} + \delta_m(t) \qquad\qquad d_t(t) = d_{t0} + \delta_t(t)$$

so we now have

$$\vec{E}_d = \vec{E}_0\big(\sin(kd_s + 2kd_{m0} + kd_d - \omega t + \phi + 2k\delta_m(t)) + \sin(kd_s + 2kd_{t0} + kd_d - \omega t + \phi + 2k\delta_t(t))\big) \tag{8}$$

Equation (8) includes terms representing oscillations on two different orders of magnitude. The variations in the target and mirror distances $\delta_t(t)$ and $\delta_m(t)$ occur with frequencies characteristic of sonic vibrations, approximately 3-5kHz. In contrast, the oscillation of the laser, represented by ω, will be at least several GHz for a maser and nearly 10^{15}Hz for the visible-light lasers that will be available in practice. Therefore we can treat $\delta_m(t)$ and $\delta_t(t)$ as approximately constant over a few oscillations of the laser radiation. Accordingly, we define our zero point of time and the quantities d_{m0} and d_{t0} such that

$$kd_s + 2kd_{m0} + kd_d - \omega t + \phi \qquad \text{and} \qquad kd_s + 2kd_{t0} + kd_d - \omega t + \phi$$

are integral multiples of 2π. This reduces equation (8) to

$$\vec{E}_d(t) = \vec{E}_0\big(\sin(2k\delta_m(t)) + \sin(2k\delta_t(t))\big) \tag{9}$$

1.3 Optimizing Intensity Variation

Our detector measures the electromagnetic intensity, given by

$$I(t) \propto E_d^2(t) \tag{10}$$

$$I(t) \propto E_0^2\big(\sin(2k\delta_m(t)) + \sin(2k\delta_t(t))\big)^2 \tag{11}$$

Differentiating $I(t)$, we find that for a small change in $\delta_t(t)$ the intensity variation is

$$\frac{\partial I}{\partial \delta_t} \propto kE_0^2\big(\sin(2k\delta_m(t)) + \sin(2k\delta_t(t))\big)\cos(2k\delta_t(t)) \tag{12}$$

We would like to set up the interferometer to maximize the magnitude of $\frac{\partial I}{\partial \delta_t}$ and thus provide the greatest detectable signal, so we set

$$\frac{\partial^2 I}{\partial \delta_m \partial \delta_t} \propto 8k^2E_0^2\cos(2k\delta_m(t))\cos(2k\delta_t(t)) = 0$$

$$\cos(2k\delta_m(t)) = 0 \qquad \text{or} \qquad \cos(2k\delta_t(t)) = 0 \tag{13}$$

The left condition leads to the maximum variation, while the right condition leads to a minimum of zero variation (which would render the apparatus useless). Of course, this result is not really practical since we can't regulate the position $\delta_t(t)$ to that accuracy, but it is useful for the next section.

1.4 Feedback Control

This last result presents an immediate problem: if the function $\cos(2k\delta_t(t))$ becomes equal to zero, the intensity $I(t)$ reaches a local extremum and it will be impossible to tell which way it comes back — that is, when the target distance changes in such a way as to extremize $I(t)$, there is no way to tell whether the target keeps moving in the same direction or switches direction, since both cases lead to the same change in intensity.

[1] These δs represent a small changing component of the path length, not the Dirac or Kronecker delta functions, which are not used anywhere in this document.

However, equation (9) suggests an alternate method: we can effectively measure $\delta_t(t)$ by altering $\delta_m(t)$ in such a way as to keep $I(t)$ roughly constant. This requires that

$$\frac{\mathrm{d}I}{\mathrm{d}t} \propto \frac{\partial I}{\partial \delta_m}\frac{\mathrm{d}\delta_m}{\mathrm{d}t} + \frac{\partial I}{\partial \delta_t}\frac{\mathrm{d}\delta_t}{\mathrm{d}t} = 0$$

or

$$4kE_0^2\big(\sin(2k\delta_m(t)) + \sin(2k\delta_t(t))\big)\left(\cos(2k\delta_m(t))\frac{\mathrm{d}\delta_m}{\mathrm{d}t} + \cos(2k\delta_t(t))\frac{\mathrm{d}\delta_t}{\mathrm{d}t}\right) = 0 \tag{14}$$

We can make this zero by maintaining one of these conditions:

$$\delta_m(t) = -\delta_t(t) \qquad \text{or} \qquad \delta_m(t) = \delta_t(t) + \pi \tag{15}$$

(perhaps in addition to other, unnecessarily complicated ways). Essentially, the adjustments we make to the piezo mirror to keep the intensity constant will duplicate (up to a sign) the movements of the target.

In practice, we cannot assume the existence of a perfect feedback system, and there is some possibility that a sudden jump in $\delta_t(t)$ will knock it significantly out of sync with $\delta_m(t)$. However, because the sine and cosine functions are cyclic, we can alter $\delta_m(t)$ in either direction in such a case and it will reach a point that restores the equilibrium before too long. This does present complications with the electronics, though, as described below.

2 Construction

2.1 The Interferometer Setup

There were a couple of practical issues with our interferometer setup that we didn't anticipate, which required making minor changes to the design.

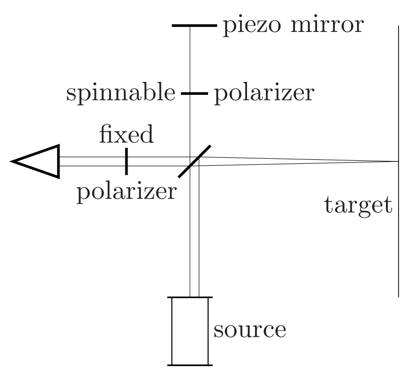

Figure 2: Schematic of our actual interferometer setup. This differs from the basic setup (figure (1.1)) only in the addition of the two polarizers and the contraction of the target distance.

The primary problem was that the intensities of the reflected and local beams would be drastically different, since the local beam was bouncing off a full mirror, whereas the reflected beam was bouncing off a partially reflective surface (i.e. glass or plastic). This asymmetry makes it harder to get a usable interference pattern. We considered various options for attenuating the local beam before settling on the obvious choice of polarizers.[2] As it turns out, the laser beam is already polarized, so by passing the local beam through a polarizer oriented at some angle to the polarization of the beam, we could block any desired percentage, simply by altering the angle.

However, this presented a slight problem: the local and reflected beams had to be of the same polarization to produce a satisfactory interference pattern. We solved this by putting a second, fixed polarizer in between the beam splitter and the detector, so that both the local and reflected beams would pass through it. We oriented this polarizer such that it would allow the maximum possible fraction of the reflected beam through, but would block the perpendicular component of the local beam. This not only guaranteed that both beams would have the same polarization, but it also helped to further attenuate the local beam.

[2]Thanks to Greg Snyder for that inspiration

Another issue we encountered was that of the coherence length, which is the maximum length over which a laser can reasonably be expected to maintain a coherent phase. Random fluctuations in the internal workings of the laser will cause essentially spontaneous phase changes over longer periods. The difficulty here was that we needed our two path lengths to be the same to the order of the coherence length, in order to get a meaningful interference pattern. We were able to work around this in the lab by setting the two path lengths equal manually, but for an actual, portable version of the device, we would need either a laser with a very long coherence length, or some method of altering the local path length.

2.2 The Feedback Circuit

The overall logical layout of the system is in figure (3). Basically, the preamp converts the current output of the detector (proportional to intensity) into a voltage, which we then feed into the difference amplifier. We subtract off a set voltage in the difference amplifier to get an error signal which averages out to zero, then we can run that error signal through the feedback loops, so that an error signal produces a response in the piezo (the P controller) and also to try and damp out random, high-frequency fluctuations (the I controller).

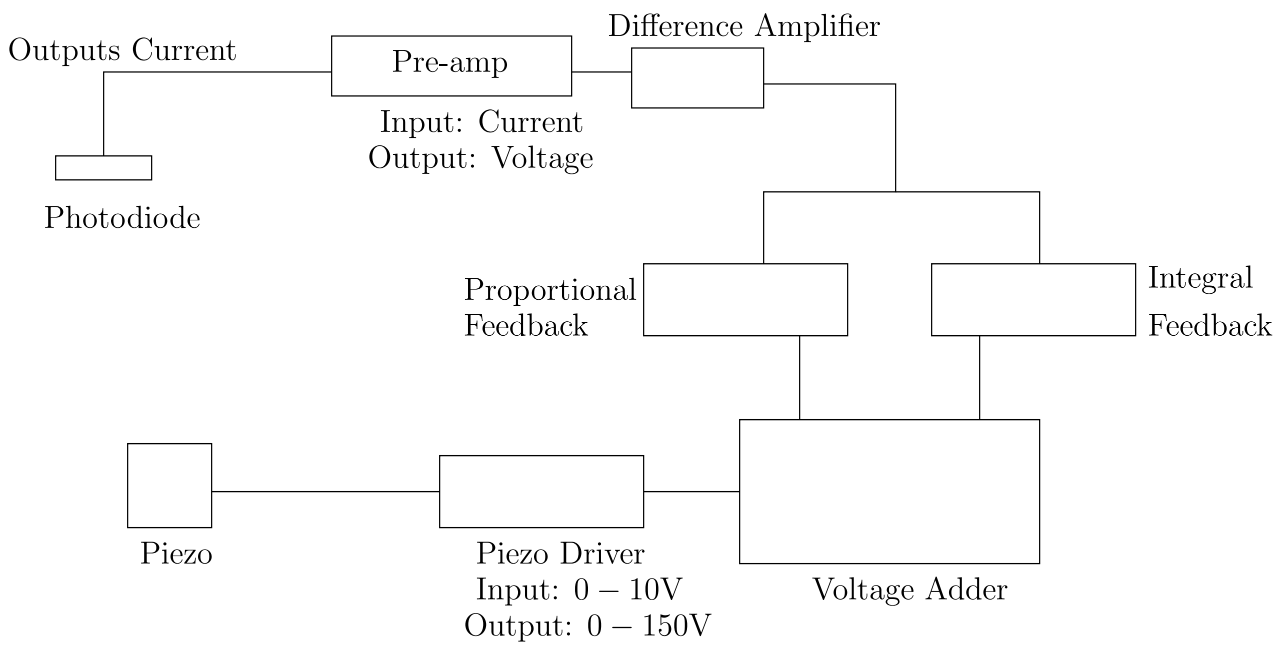

Figure 3: Global View

The feedback circuit is necessary because, as detailed above, it is possible that the position of the target will change by more than a wavelength of the light. Without feedback, if that happens, it will cause us to lose our lock. Instead, the feedback is set up so that the if an error signal develops, the circuit will drive the piezo to compensate for the position change, so that the error signal goes back to 0.

2.2.1 Difference Amplifier

The difference amplifier, which computes the error signal from the input voltage and a set point, is in figure (4). All the resistances were chosen to be about equal; the resistances shown are those we actually measured with the multimeter. We would control the set point by tweaking the potentiometer so that the output signal averaged to approximately 0V. That way, we wouldn't get any large error buildup in the I controller.

2.2.2 Proportional Feedback

Our proportional feedback circuit is displayed in figure (5). This is the component that is primarily responsible for implementing the negative feedback we require, which serves to drive the system back toward equilibrium.

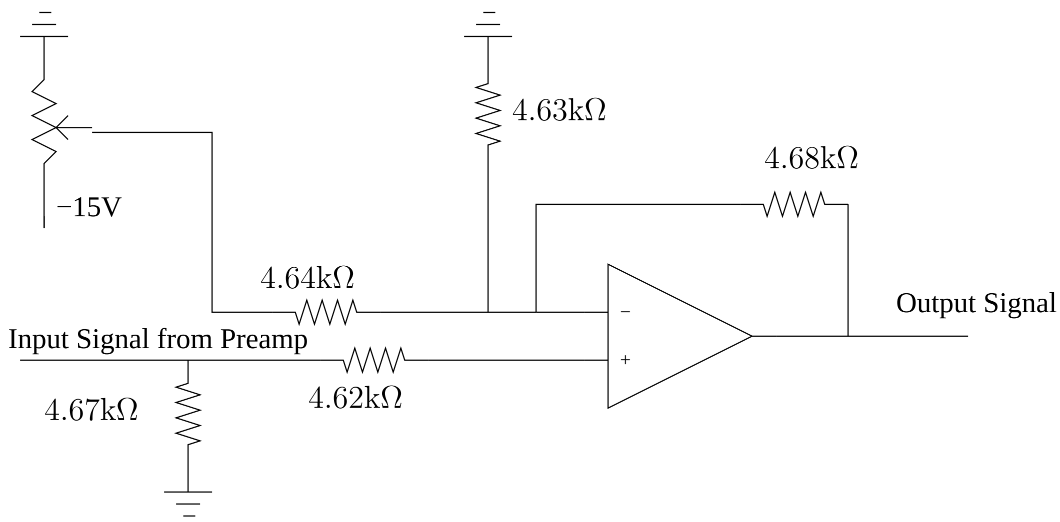

Figure 4: Difference Amplifier

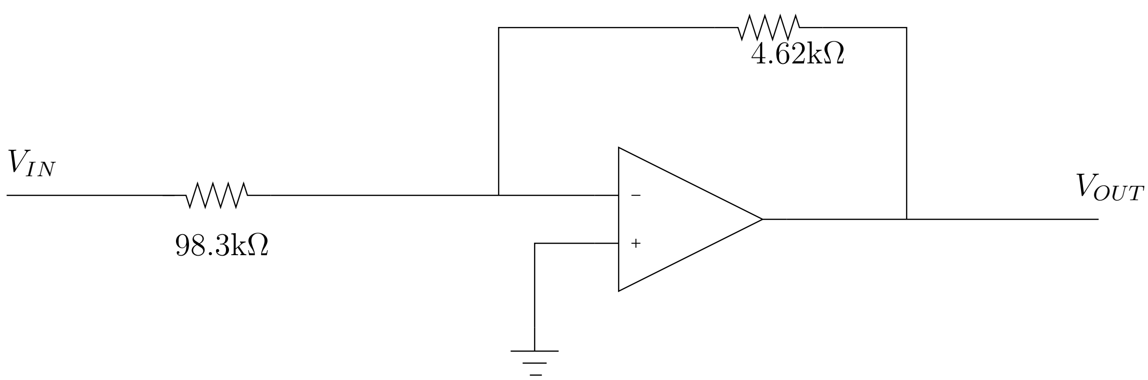

Figure 5: Proportional Feedback Circuit

The behavior of this circuit element can be described by the equation

$$V_{OUT} = -\frac{4.62\text{k}\Omega}{98.3\text{k}\Omega} V_{IN} \tag{16}$$

In this case, our P controller actually reduces, rather than amplifies, the voltage input. This became necessary due to the large gain in the preamp, and we needed to reduce the voltage in order to make it fit the input parameters of the piezo driver.

2.2.3 Integral Feedback

We also designed an integral feedback loop to cancel out random noise due to factors such as the wavelength dispersion of the laser. The layout is described in figure (6).

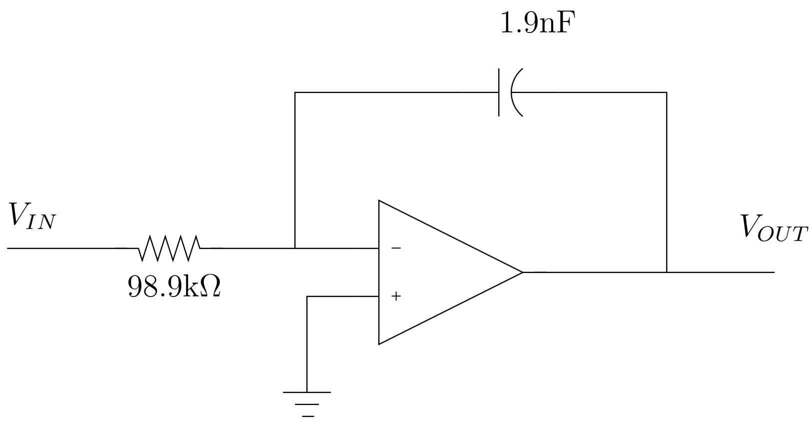

Figure 6: Integral Feedback Circuit

The operation of this element is described by the equation

$$V_{OUT} = \frac{1}{98.9\text{k}\Omega * 1.9\text{nF}} \int V_{IN} dt \tag{17}$$

Essentially, the output voltage is the integral of the input voltage over time (restarting whenever the capacitor is discharged), multiplied by $(RC)^{-1}$.

2.2.4 Voltage Adder

We needed to add the integral and proportional feedback using the component in figure 2.2.4.

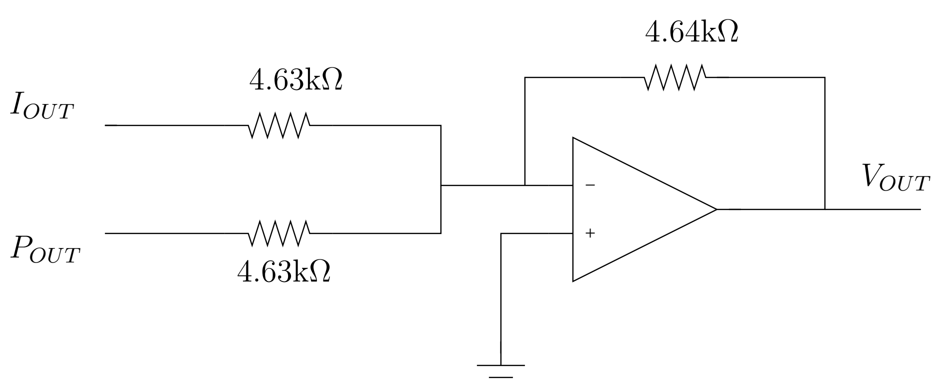

Figure 7: Voltage adder

As with the difference amplifier, we tried to choose all resistances approximately equal, so that $V_{OUT} \approx I_{OUT} + P_{OUT}$.

2.2.5 Power Supply Smoothing

There is one more issue with our circuit: fluctuations in the power supply could make the behaviour of the opamps unreliable. So, we attached capacitors to the power supplies of all opamps, as in figure (8). This way,

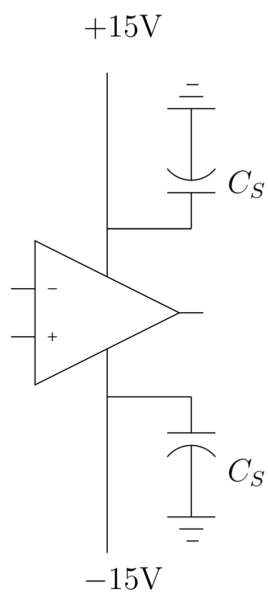

Figure 8: Op-Amp Power Supply

the capacitor tends to smooth out the power supply to the op-amps, and so they don't introduce noise into the system.

2.3 Recording

All this complicated circuitry creates an interesting problem — where do we record the signal? We have two options: we can record the error signal that gets input into the P and I controllers, or we can record the output of the P and the I controllers (the input of the piezo driver). If the feedback has a really high gain, then the error signal will always be very small (because that's what feedback circuits do), so it would be best to record the signal at the output of the P and the I controllers. On the other hand, if the gain is relatively small, then then the error signal will be larger compared to the output of the PI controller, and so you would want to read the error signal.

Through testing, we found that our gain was in the mid-range, so for a better measurement, we'd need either a lower gain (and then measure the error signal) or a higher gain (and then measure the PI output). But the error signal still gave us clearer recording.

3 Future Modifications

3.1 Reset Circuit

In general, we can't expect to fix our error signal perfectly at zero, which means that charge will build up on the capacitor over time, reducing its effectiveness. We'd like to design a reset circut so that, once the capacitor in the I controller gets above or below a certain voltage, it will discharge the capacitor. The circuitry would look something like figure (9). Here, the output of the PI controller is fed into a couple of different op-amps that

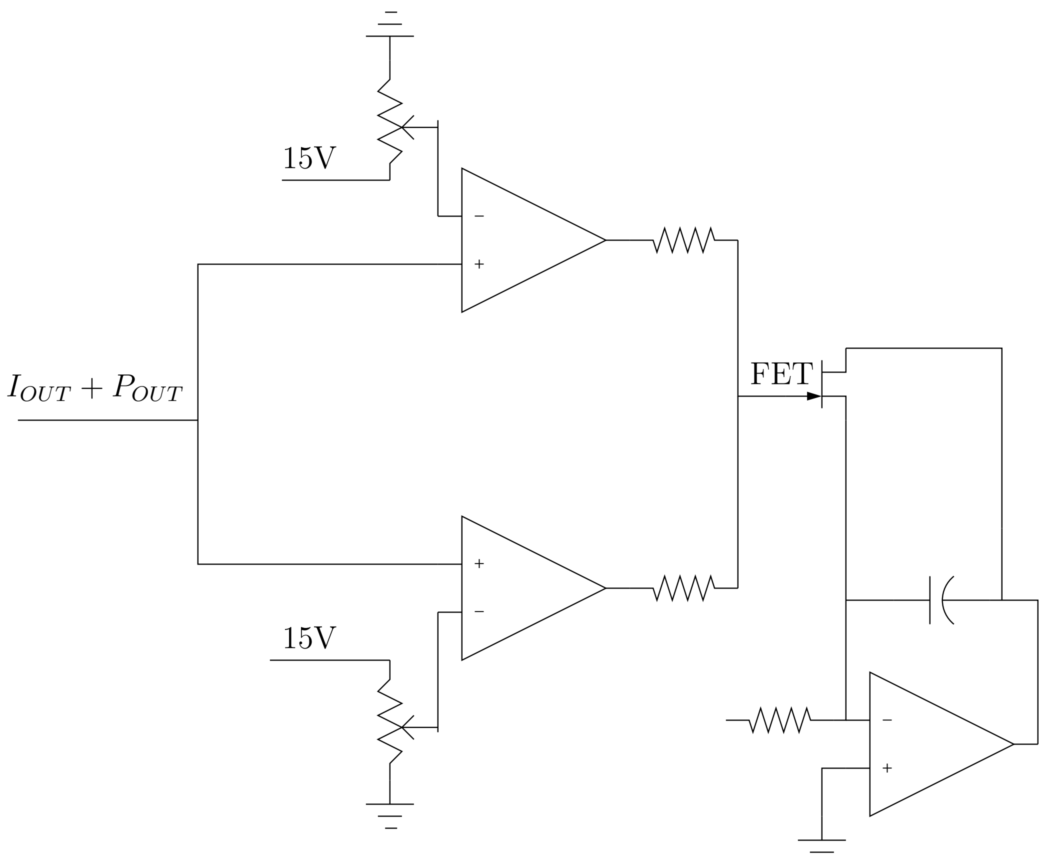

Figure 9: Reset Circuit with the I controller

are basically difference amplifiers. The potentiometers can be set such that whenever the voltage goes above a chosen V_{MAX} or below a chosen V_{MIN} it will create a positive voltage output on one of the op-amps, and that will open the FET and discharge the capacitor in the I controller.

3.2 Portability

One of our original hopes was to build the apparatus such that it could be packaged in a small box and taken around to use anywhere. However, we were unable to do so with the materials available and in the time we had. Now that the circuitry and the required parameters of the device have been established, assembling a portable version seems like a reasonable goal if/when we can come back to this project.

4 Credits

We would like to thank Professor Romalis and Georgios for their tireless assistance and infinite wisdom (we measured a lower bound of 10^{44}), and the Princeton University Physics Department for so readily indulging our fantasies.