ONE-WAY RADAR EQUATION / RF PROPAGATION

The one-way (transmitter to receiver) radar equation is derived in this section. This equation is most commonly used in RWR or ESM type of applications. The following is a summary of the important equations explored in this section:

ONE-WAY RADAR EQUATION

Peak Power at Receiver Input, $P_r\ (or\ S) = P_D A_e = \frac{P_t G_t A_e}{4\pi R^2}$ *and Antenna Gain,* $G = \frac{4\pi A_e}{\lambda^2}$ *or: Equivalent Area,* $A_e = \frac{G\lambda^2}{4\pi}$

So the one-way radar equation is :

$$S\ (or P_r) = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} = P_t G_t G_r \left[\frac{c^2}{(4\pi f R)^2}\right]^* \qquad (Note:\ \lambda = \frac{c}{f})$$

* keep λ, c, and R in the same units

On reducing to log form this becomes:

$10\log P_r = 10\log P_t + 10\log G_t + 10\log G_r - 20\log fR + 20\log(c/4\pi)$

or in simplified terms:

$10\log P_r = 10\log P_t + 10\log G_t + 10\log G_r - \alpha_1$ (in dB)

Where: α_1 = one-way free space loss = $20\log(f_1 R) + K_1$ (in dB)

and: $K_1 = 20\log[(4\pi/c)$(Conversion factors if units if not in m/sec, m, and Hz)]

Note: To avoid having to include additional terms for these calculations, always combine any transmission line loss with antenna gain

Values of K_1 (in dB)

Range (units)	f_1 in MHz K_1 =	f_1 in GHz K_1 =
NM	37.8	97.8
km	32.45	92.45
m	-27.55	32.45
yd	-28.33	31.67
ft	-37.87	22.13

Note: Losses due to antenna polarization and atmospheric absorption (Sections 3-2 & 5-1) are not included in any of these equations.

Recall from Section 4-2 that the power density at a distant point from a radar with an antenna gain of G_t is the power density from an isotropic antenna multiplied by the radar antenna gain.

Power density from radar, $P_D = \frac{P_t G_t}{4\pi R^2}$ **[1]**

If you could cover the entire spherical segment with your receiving antenna you would theoretically capture all of the transmitted energy. You can't do this because no antenna is large enough. (A two degree segment would be about a mile and three-quarters across at fifty miles from the transmitter.)

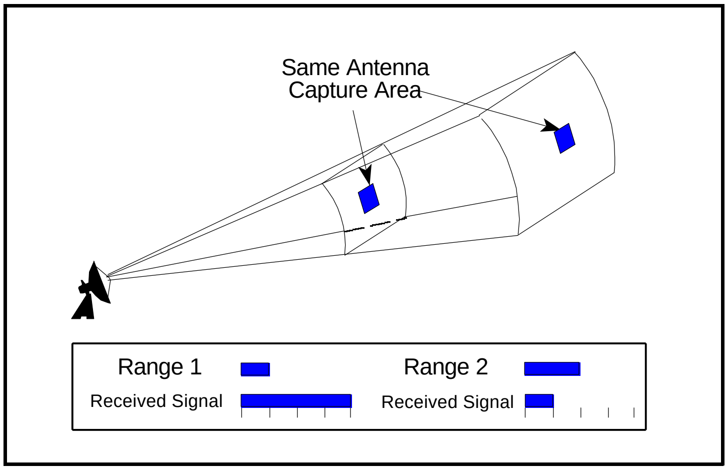

Figure 1. Power Density vs. Range

A receiving antenna captures a portion of this power determined by it's effective capture Area (A_e). The received power available at the antenna terminals is the power density times the effective capture area (A_e) of the receiving antenna.

For a given receiver antenna size the capture area is constant no matter how far it is from the transmitter, as illustrated in Figure 1. This concept is shown in the following equation:

$$P_R \text{ (or S)} = P_e = \frac{P_t G_t A_e}{4\pi R^2} \quad \text{which is known as the one-way (beacon) equation}$$

In order to maximize energy transfer between an antenna and transmitter or receiver, the antenna size shoul correlate λ/4. Control o beamwidth shape may become a problem when the size of the active element exceeds several wavelengths.

Th relation between an antenna's effectiv capture area (A_e is:

$$Antenna\ Gain,\ G = \frac{4\pi A_e}{\lambda^2}$$

$$or:\ Equivalent\ Area,\ A_e = \frac{G\lambda^2}{4\pi} \qquad \textbf{[4]}$$

effective aperture is in units of length squared, s proport wavelength. This physically means that to maintain the gain when doubling the frequency, the area i reduced by 1/4. This concept is illustrated in Figure 2.

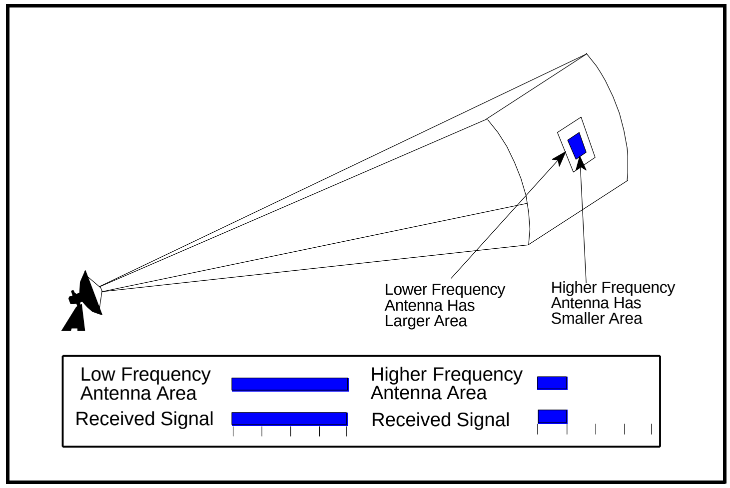

Figure 2. Capture Area vs Frequency

If equation [4] is substituted into equation [2], the following relationship results:

$$Peak\ Power\ at\ Receiver\ Input = S\ (or\ P_R) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 R^2} = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} \qquad \textbf{[5]}$$

is the signal calculated one-way from a transmitter to a receiver. For instance, a radar application might be to rmine the signal received by a RWR, ESM, or an ELINT receiver. It is a general purpose equation and could be

The free space travel of radio waves can, of course, be blocked, reflected, or distorted by objects in their path such

As received signal power decreases by 1/4 (6 dB). This is due to the [2] term in equation [5].

illust a square on radius is decreased by 1/2, you further blow up the balloon, so the diameter or radius i doubled, the square has quadrupled in area.

ONE WAY SIGNAL STRENGTH (S)

S ⬇ **6 dB** (1/4 pwr)	**S** decreases by **6 dB** when the distance doubles	**2R** ⬆ **R**
6 dB (4x pwr) ⬆ **S**	**S** increases by **6 dB** when the distance is half	**R** ⬇ **0.5 R**

The one-way free space loss factor (α_1), (sometimes called the path loss factor) is given by the term $(4\pi R^2)(4\pi/\lambda^2)$ or $(4\pi R/\lambda)^2$. As shown in Figure 3, the loss is due to the ratio of two factors (1) the effective radiated area of the transmit antenna, which is the surface area of a sphere ($4\pi R^2$) at that distance (R), and (2) the effective capture area (A_e) of the receive antenna which has a gain of one. If a receiving antenna could capture the whole surface area of the sphere, there would be no spreading loss, but a practical antenna will capture only a small part of the spherical radiation. Space loss is calculated using isotropic antennas for both transmit and receive, so α_1 is independent of the actual antenna. Using Gr = 1 in equation [11] in section 3-1, $A_e = \lambda^2/4\pi$. Since this term is in the denominator of α_1, the higher the frequency (lower λ) the more the space loss. Since G_t and G_r are part of the one-way radar equation, S (or P_r) is adjusted according to actual antennas as shown in the last portion of Figure 3. The value of the received signal (S) is:

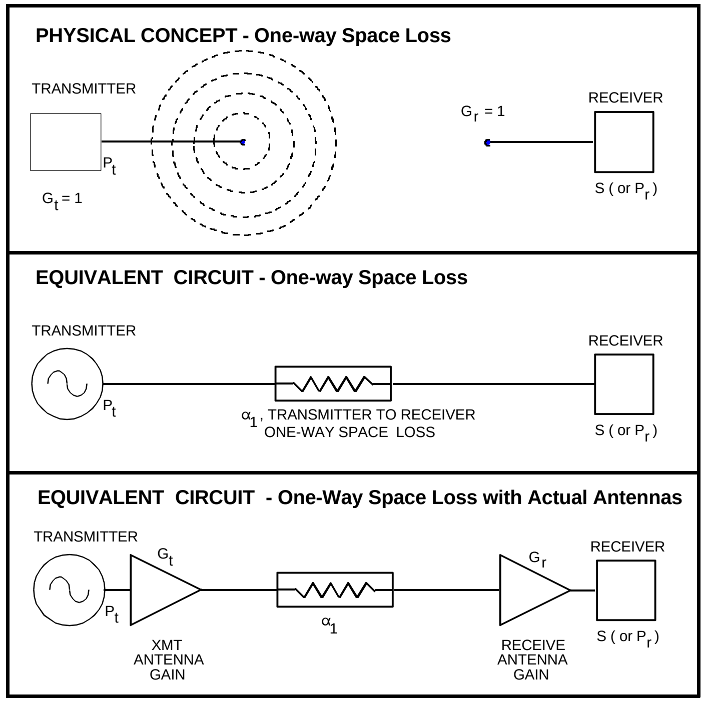

Figure 3. Concept of One-Way Space Loss

$$S\ (or\ P_R) = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} = P_t G_t G_r \left[\frac{\lambda^2}{(4\pi R)^2}\right] \qquad \textbf{[6]}$$

To convert this equation to dB form, it is rewritten as:

$$10\log(S\ or P_r) = 10\log(P_t G_t G_r) + 20\log\left[\frac{\lambda}{4\pi R}\right]^* \qquad (*\ keep\ \lambda\ and\ R\ in\ same\ units) \qquad \textbf{[7]}$$

Since $\lambda = c / f$, equation [7] can be rewritten as:

$$10\ \mathrm{Log}\ (S\ \mathrm{or}\ P_r) = 10\ \mathrm{Log}(P_t G_t G_r) - \alpha_1 \qquad \textbf{[8]}$$

Where the one-way free space loss, α_1, is defined as: $\alpha_1 = 20 Log\left[\frac{4\pi f R}{c}\right] *$ **[9]**

The signal received equation in dB form is: $10\log (P_r \text{ or } S) = 10\log P_t + 10\log G_t + 10\log G_r - \alpha_1$ **[10]**

The one-way free space loss, α_1, can be given in terms of a variable and constant term as follows:

$$\alpha_1 = 20 Log\left[\frac{4\pi f R}{c}\right]^* = 20 Log\ f_1\ R + K_1 \qquad (in\ dB) \qquad \textbf{[11]}$$

The value of f_1 can be either in MHz or GHz as shown with commonly used units of R in the adjoining table.

$$where\ K_1 = 20 Log\left[\frac{4\pi}{c}\cdot(Conversion\ units\ if\ not\ in\ m/\mathrm{sec},\ m,\ and\ Hz)\right]$$

Values of K_1 (dB)		
Range (units)	f_1 in MHz K_1 =	f_1 in GHz K_1 =
NM	37.8	97.8
km	32.45	92.45
m	-27.55	32.45
yd	-28.33	31.67
ft	-37.87	22.13

Note: To avoid having to include additional terms for these calculations, always combine any transmission line loss with antenna gain.

A value for the one-way free space loss (α_1) can be obtained from:

(a) The One-way Free Space Loss graph (Figure 4). Added accuracy can be obtained using the Frequency Extrapolation graph (Figure 5)

(b) The space loss nomograph (Figure 6 or 7)

(c) The formula for α_1, equation [11].

FOR EXAMPLE:

Find the value of the one-way free space loss, α_1, for an RF of 7.5 GHz at 100 NM.

(a) From Figure 4, find 100 NM on the X-axis and estimate where 7.5 GHz is located between the 1 and 10 GHz lines (note dot). Read α_1 as 155 dB. An alternate way would be to read the α_1 at 1 GHz (138 dB) and add the frequency extrapolation value (17.5 dB for 7.5:1, dot on Figure 5) to obtain the same 155 dB value.

(b) From the nomogram (Figure 6), the value of α_1 can be read as 155 dB (Note the dashed line).

(c) From the equation 11, the precise value of α_1 is 155.3 dB.

Remember, α_1 is a free space value. If there is atmospheric attenuation because of absorption of RF due to certain molecules in the atmosphere or weather conditions etc., the atmospheric attenuation is in addition to the space loss (refer to Section 5-1).

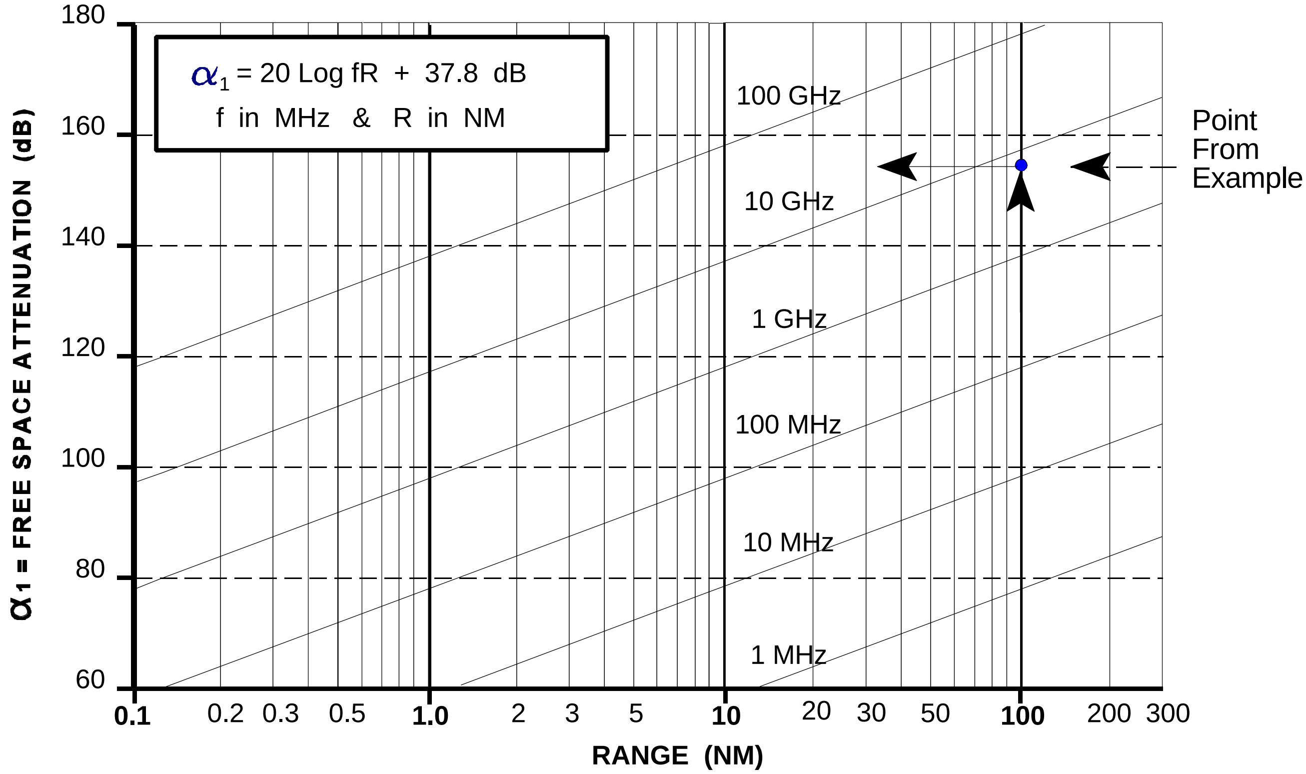

Figure 4. One-Way Free Space Loss

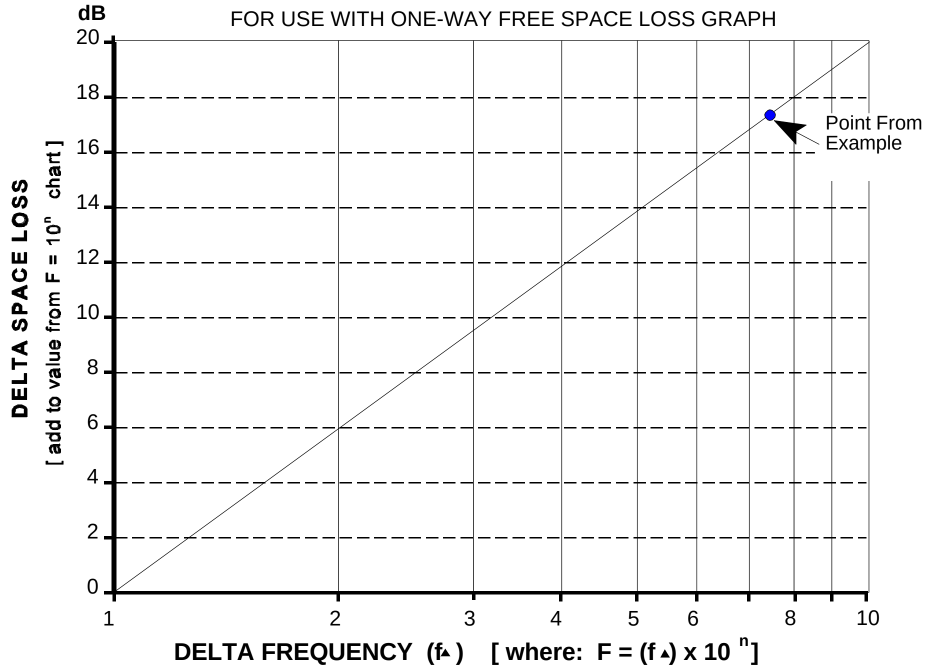

Figure 5. Frequency Extrapolation

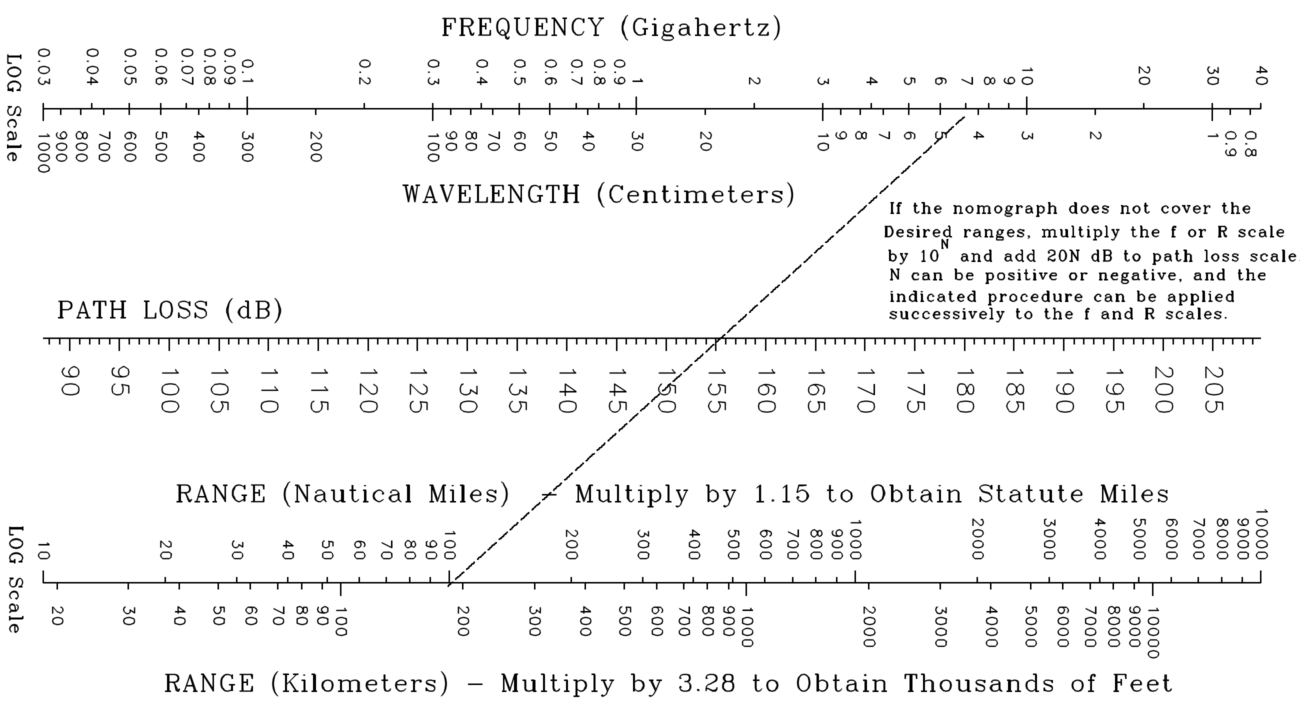

Figure 6. One-Way Space Loss Nomograph For Distances Greater Than 10 Nautical Miles

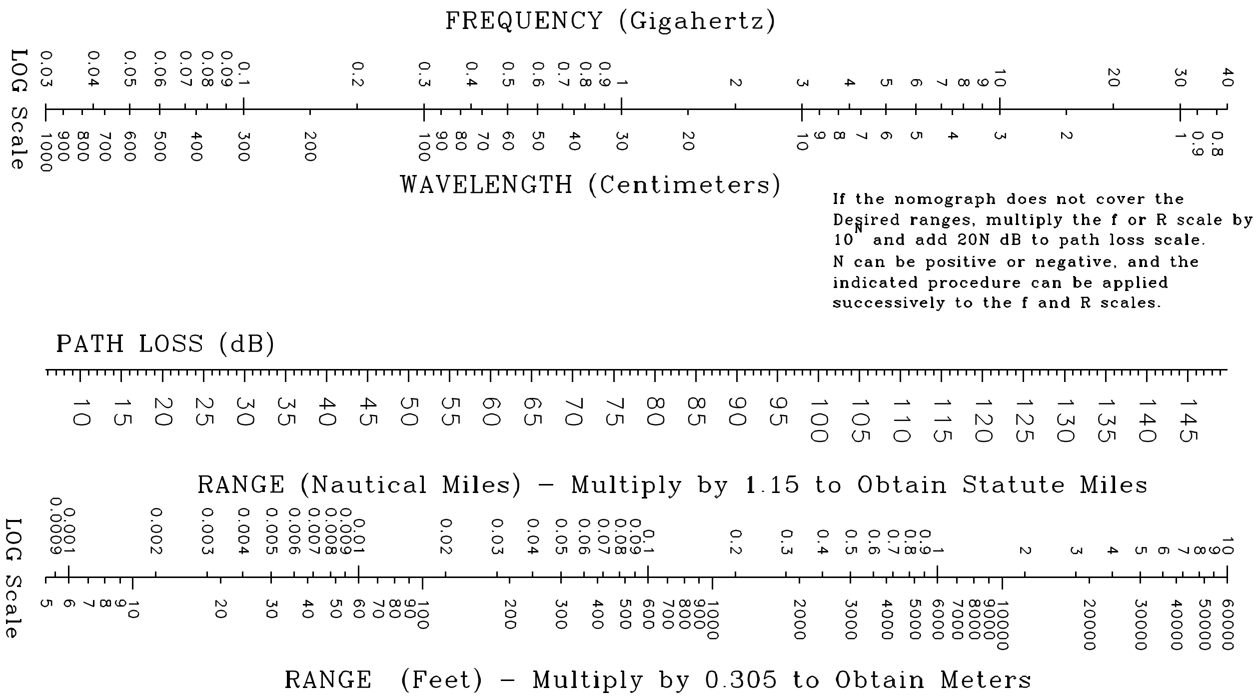

Figure 7. One-Way Space Loss Nomograph For Distances Less Than 10 Nautical Miles

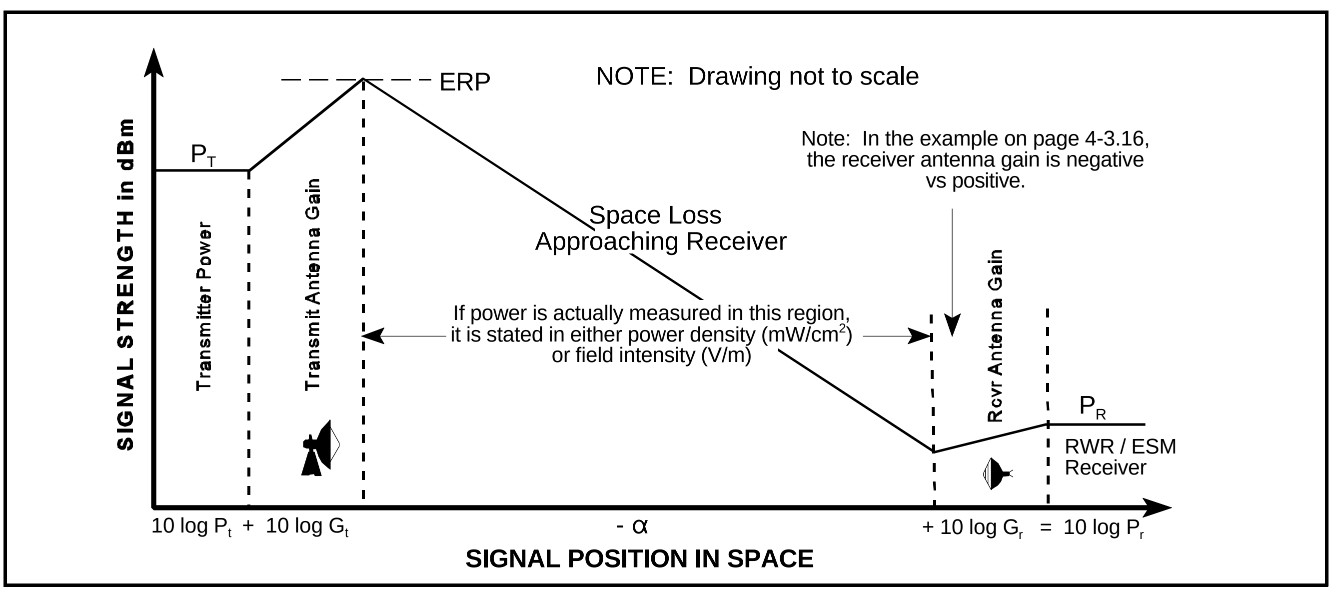

Figure 8. Visualization of One-Way Radar Equation

Figure 8 is the visualization of the losses occurring in one-way radar equation. Note: To avoid having to include additional terms, always combine any transmission line loss with antenna gain. Losses due to antenna polarization and atmospheric absorption also need to be included.

RWR/ESM RANGE EQUATION (One-Way)

The one-way radar (signal strength) equation [5] is rearranged to calculate the maximum range R_{max} of RWR/ESM receivers. It occurs when the received radar signal just equals S_{min} as follows:

$$R_{max} \cong \left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 S_{min}}\right]^{\frac{1}{2}} \quad or \quad \left[\frac{P_t G_t G_r c^2}{(4\pi f)^2 S_{min}}\right]^{\frac{1}{2}} \quad or \quad \left[\frac{P_t G_t A_e}{4\pi S_{min}}\right]^{\frac{1}{2}} \qquad \textbf{[12]}$$

In log form:

$$20\log R_{max} = 10\log P_t + 10\log G_t - 10\log S_{min} - 20\log f + 20\log(c/4\pi) \qquad \textbf{[13]}$$

and since $K_1 = 20\log\{4\pi/c$ times conversion units if not in m/sec, m, and Hz$\}$ (Refer to section 4-3 for values of K_1).

$$10\log R_{max} = \tfrac{1}{2}[\,10\log P_t + 10\log G_t - 10\log S_{min} - 20\log f - K_1] \qquad \text{(keep } P_t \text{ and } S_{min} \text{ in same units)} \qquad \textbf{[14]}$$

If you want to convert back from dB, then $R_{max} \cong \frac{10^{MdB}}{20}$, where M dB is the resulting number in the brackets of equation 14.

From Section 5-2, Receiver Sensitivity / Noise, S_{min} is related to the noise factor S: $S_{min} = (S/N)_{min}\,(NF)KT_oB$ **[15]**

The one-way RWR/ESM range equation becomes:

$$R_{max} \cong \left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 (S/N)_{min}(NF)KT_oB}\right]^{\frac{1}{2}} \quad or \quad \left[\frac{P_t G_t G_r c^2}{(4\pi f)^2 (S/N)_{min}(NF)KT_oB}\right]^{\frac{1}{2}} \quad or \quad \left[\frac{P_t G_t A_e}{4\pi (S/N)_{min}(NF)KT_oB}\right]^{\frac{1}{2}} \qquad \textbf{[16]}$$

RWR/ESM RANGE INCREASE AS A RESULT OF A SENSITIVITY INCREASE

As shown in equation [12] $S_{min}^{-1} \propto R_{max}^2$ Therefore, $-10\log S_{min} \propto 20\log R_{max}$ and the table below results:

% Range Increase: Range + (% Range Increase) x Range = New Range

i.e., for a 6 dB sensitivity increase, 500 miles +100% x 500 miles = 1,000 miles

Range Multiplier: Range x Range Multiplier = New Range i.e., for a 6 dB sensitivity increase 500 miles x 2 = 1,000 miles

dB Sensitivity Increase	% Range Increase	Range Multiplier	dB Sensitivity Increase	% Range Increase	Range Multiplier
+ 0.5	6	1.06	10	216	3.16
1.0	12	1.12	11	255	3.55
1.5	19	1.19	12	298	3.98
2	26	1.26	13	347	4.47
3	41	1.41	14	401	5.01
4	58	1.58	15	462	5.62
5	78	1.78	16	531	6.31
6	100	2.0	17	608	7.08
7	124	2.24	18	694	7.94
8	151	2.51	19	791	8.91
9	182	2.82	20	900	10.0

RWR/ESM RANGE DECREASE AS A RESULT OF A SENSITIVITY DECREASE

As shown in equation [12] $S_{min}^{-1} \propto R_{max}^{2}$ Therefore, $-10 \log S_{min} \propto 20 \log R_{max}$ and the table below results:

% Range Decrease: Range - (% Range decrease) x Range = New Range

i.e., for a 6 dB sensitivity decrease, 500 miles - 50% x 500 miles = 250 miles

Range Multiplier: Range x Range Multiplier = New Range i.e., for a 6 dB sensitivity decrease 500 miles x .5 = 250 miles

dB Sensitivity Decrease	% Range Decrease	Range Multiplier	dB Sensitivity Decrease	% Range Decrease	Range Multiplier
- 0.5	6	0.94	-10	68	0.32
- 1.0	11	0.89	- 11	72	0.28
- 1.5	16	0.84	- 12	75	0.25
- 2	21	0.79	- 13	78	0.22
- 3	29	0.71	- 14	80	0.20
- 4	37	0.63	- 15	82	0.18
- 5	44	0.56	- 16	84	0.16
- 6	50	0.50	- 17	86	0.14
- 7	56	0.44	- 18	87	0.13
- 8	60	0.4	- 19	89	0.11
- 9	65	0.35	- 20	90	0.10

Example of One-Way Signal Strength: A 5 (or 7) GHz radar has a 70 dBm signal fed through a 5 dB loss transmission line to an antenna that has 45 dB gain. An aircraft that is flying 31 km from the radar has an aft EW antenna with -1 dB gain and a 5 dB line loss to the EW receiver (assume all antenna polarizations are the same). Note: The respective transmission line losses will be combined with antenna gains, i.e.:

-5 +45 = 40 dB, -5 - 1 = -6 dB, -10 + 5 = -5 dB.

(1) What is the power level at the input of the EW receiver?

Answer (1): P_r at the input to the EW receiver = Transmitter power - xmt cable loss + xmt antenna gain - space loss + rcvr antenna gain - rcvr cable loss.

Space loss (from section 4-3) @ 5 GHz = $20 \log f R + K_1$ = 20 log (5x31) + 92.44 = 136.25 dB.

Therefore, P_r = 70 + 40 - 136.25 - 6 = -32.25 dBm @ 5 GHz (P_r = -35.17 dBm @ 7 GHz since α_1 = 139.17 dB)

(2) If the received signal is fed to a jammer with a gain of 60 dB, feeding a 10 dB loss transmission line which is connected to an antenna with 5 dB gain, what is the power level from the jammer at the input to the receiver of the 5 (or 7) GHz radar?

Answer (2): P_r at the input to the radar receiver = Power at the input to the EW receiver+ Jammer gain - jammer cable loss + jammer antenna gain - space loss + radar rcvr antenna gain - radar rcvr cable loss .

Therefore, P_r = -32.25 + 60 - 5 - 136.25 + 40 = -73.5 dBm @ 5 GHz. (P_r = -79.34 dBm @ 7 GHz since α_1 = 139.17 dB and P_t = -35.17 dBm).

This problem continues in section 4-4, 4-7, and 4-10.